

Polynomials

Explain how to add and subtract polynomials.

~~FOIL~~
FOIL

$$(x+2) + (x+5)$$

combine like terms

$$x+2+x+5 = 2x+7$$

Explain how to multiply two polynomials.

FOIL

multiply

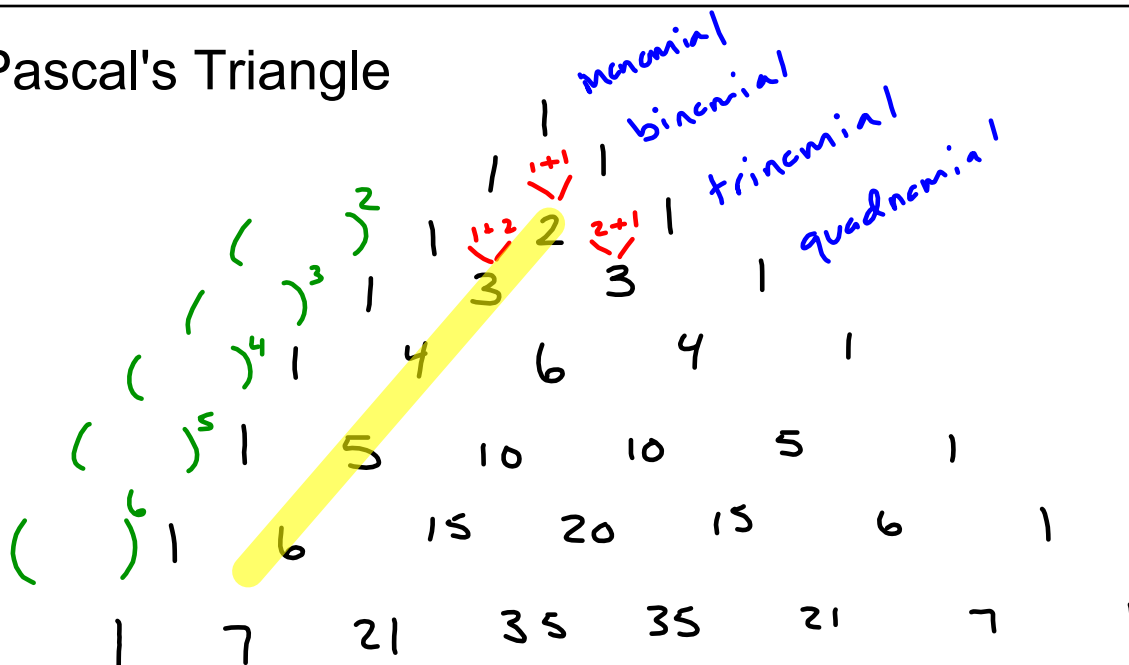
$$(x+2)(x+5)$$

$$x^2 + 5x + 2x + 10$$

$$x^2 + 7x + 10$$

Aug 7-6:57 PM

Pascal's Triangle



Aug 7-7:04 PM

Pascal's Δ

$$\begin{array}{ccccccc} & & & & 1 & & \\ & & & & 1 & & 1 \\ & & & 1 & 2 & 1 & \\ & & 1 & 3 & 3 & 1 & \\ 1 & 4 & 6 & 4 & 1 & & \end{array}$$

$(x+y)^4$

$(x+y)(x+y)(x+y)(x+y)$

$x^2+xy+xy+y^2$

$(x^2+2xy+y^2)(x^2+2xy+y^2)$

$x^4 + 2x^3y + x^2y^2 + 2x^2y + yx^2y^2 + 2xy^3$

$x^2y^2 + 2xy^3 + y^4$

$1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

Aug 8-8:29 AM

$(x+y)^4$

$1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

$(x+y)^4$

$1x^4 \quad 4x^3y \quad 6x^2y^2 \quad 4xy^3 \quad 1y^4$

Process

PASCAL Δ coefficients

Label order of # & variables

Evaluate

Aug 8-8:40 AM

$$(x+2)^4$$

$$1x^4(2)^0 \quad 4x^3(2)^1 \quad 6x^2(2)^2 \quad 4x(2)^3 \quad 1x(2)^4$$

$$x^4 + 4x^3 \cdot 2 + 6x^2 \cdot 4 + 4x \cdot 8 + 1 \cdot 1 \cdot 16$$

$$x^4 + 8x^3 + 24x^2 + 32x + 16$$

Aug 8-8:44 AM

$(x+2)^7$

More practice section 3-3 textbook


() ⁷	1	7	21	35	35	21	7	1
() ⁶	1	6	15	20	15	6	1	
() ⁵	1	5	10	10	5	1		
() ⁴	1	4	6	4	1			

$$1x^7(2)^0 \quad 7x^6(2)^1 \quad 21(x)(2)^2 \quad 35x^4(2)^3 \quad 35x^3(2)^4 \quad 21(x^2)(2)^5 \quad 7(x)(2)^6 \quad 1(x)^7(2)^7$$

$$1 \cdot x^7 \cdot 1 \quad 7 \cdot x^6 \cdot 2 \quad 21 \cdot x^5 \cdot 4 \quad 35 \cdot x^4 \cdot 8 \quad 35 \cdot x^3 \cdot 16 \quad 21 \cdot x^2 \cdot 32 \quad 7 \cdot x \cdot 64 \quad 1 \cdot 1 \cdot 128$$

$$x^7 + 14x^6 + 84x^5 + 280x^4 + 560x^3 + 672x^2 + 448x + 128$$

Aug 8-8:50 AM



Polynomials

Explain how to add and subtract polynomials.

~~FOIL~~ $(x+3) + (x+5)$ *combine like terms*
 $x+3 + x + 5 = 2x+8$

Explain how to multiply two polynomials.

$(x+3)(x+5)$

what if ... $x^2 + 5x + 3x + 15$
 $x^2 + 8x + 15$

$(x+3) - (x+5)$
 $x+3 - x - 5 = -2$

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$(x+2)^4$

$(x+2)(x+2)(x+2)(x+2)$
 $x^2 + 2x + 2x + 4$

$(x^2 + 4x + 4)(x^2 + 4x + 4)$

$x^4 + 4x^3 + 4x^2 + 4x^3 + 16x^2 + 16x$
 $+ 4x^2 + 16x + 16$

$x^4 + 8x^3 + 24x^2 + 32x + 16$

Aug 8-9:33 AM

Grading

Dok

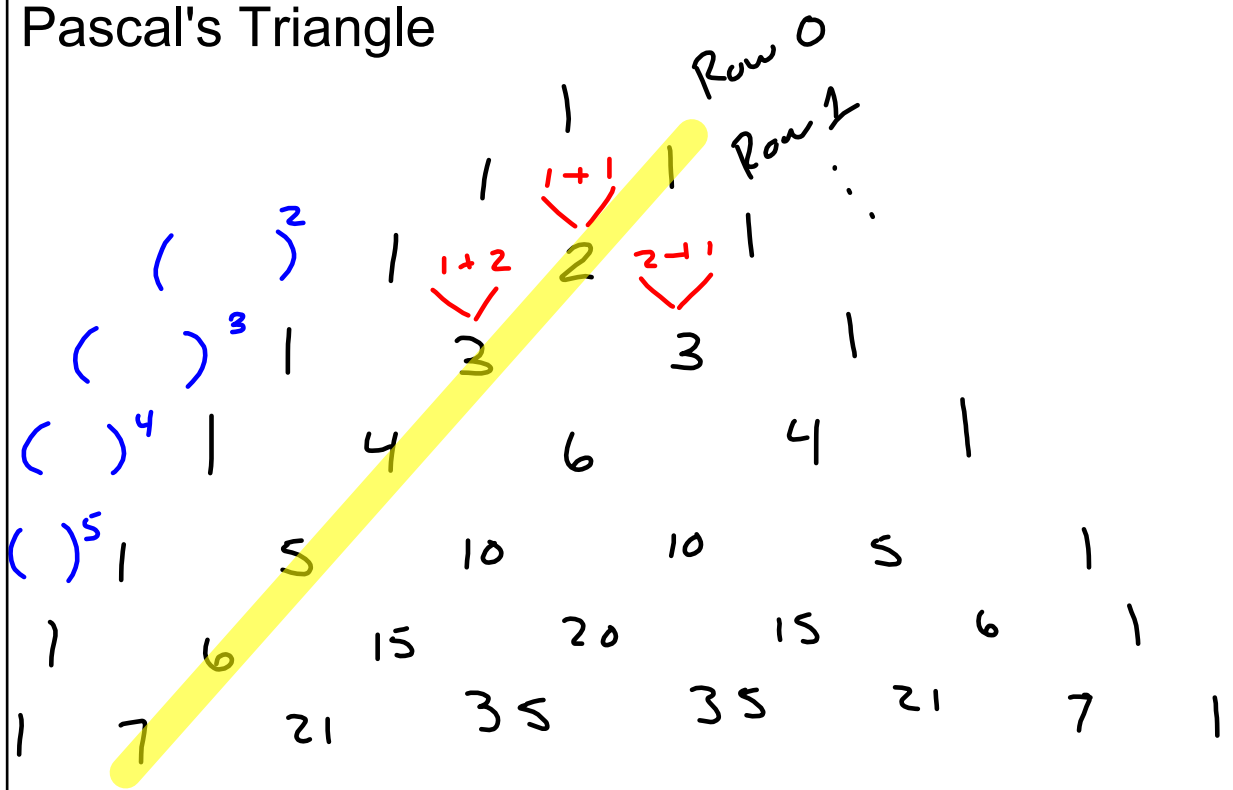
Level 1 70

Level 2 80

Level 3-4 90

Aug 8-9:25 AM

Pascal's Triangle



Aug 7-7:04 PM

$(x+y)^4$
 $[(x+y)(x+y)][(x+y)(x+y)]$
 $x^2 + xy + xy + y^2$
 $(x^2 + 2xy + y^2)(x^2 + 2xy + y^2)$
 $x^4 + 2x^3y + x^2y^2 + 2x^3y + 4x^2y^2 + 2xy^3 + x^2y^2 + 2xy^3 + y^4$
 $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

Aug 8-9:44 AM

$(x+y)^4$
PASCAL Δ

$1x^4(y)^0$	$4x^3(y)^1$	$6x^2(y)^2$	$4x^1(y)^3$	$1x^0(y)^4$
$1 \cdot x^4 \cdot 1$	$4 \cdot x^3 \cdot y$	$6 \cdot x^2 \cdot y^2$	$4 \cdot x \cdot y^3$	$1 \cdot 1 \cdot y^4$
x^4	$+ 4x^3y$	$+ 6x^2y^2$	$+ 4xy^3$	$+ y^4$

 $(x+2)^4$

Aug 8-9:51 AM

$$(x+2)^4$$

$1 \cdot x^4 \cdot 1$	$4x^3 \cdot 2$	$6x^2 \cdot 2^2$	$4x \cdot 2^3$	$1 \cdot 2^4$
x^4	$+ 8x^3$			

Aug 8-9:59 AM

Polynomials



Explain how to add and subtract polynomials.



$$x+2 + x+2 = 2x+4$$

combine like terms

$$(x+2) + (x+2)$$

Explain how to multiply two polynomials.

$$(x+2) - (x+2)$$

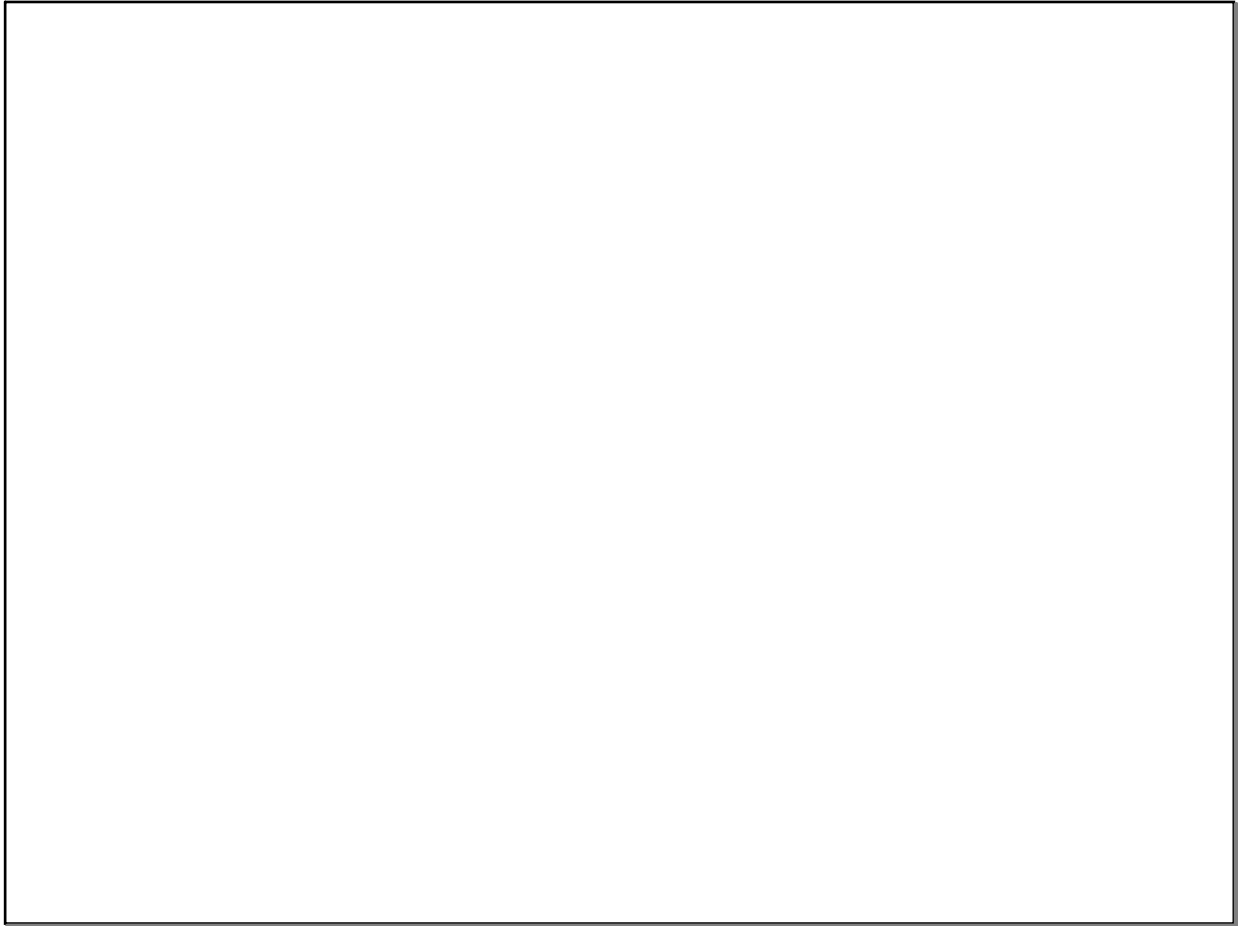
$$x+2 - x-2 = 0$$

$$(x+2)(x+2) \quad * \text{FOIL}$$

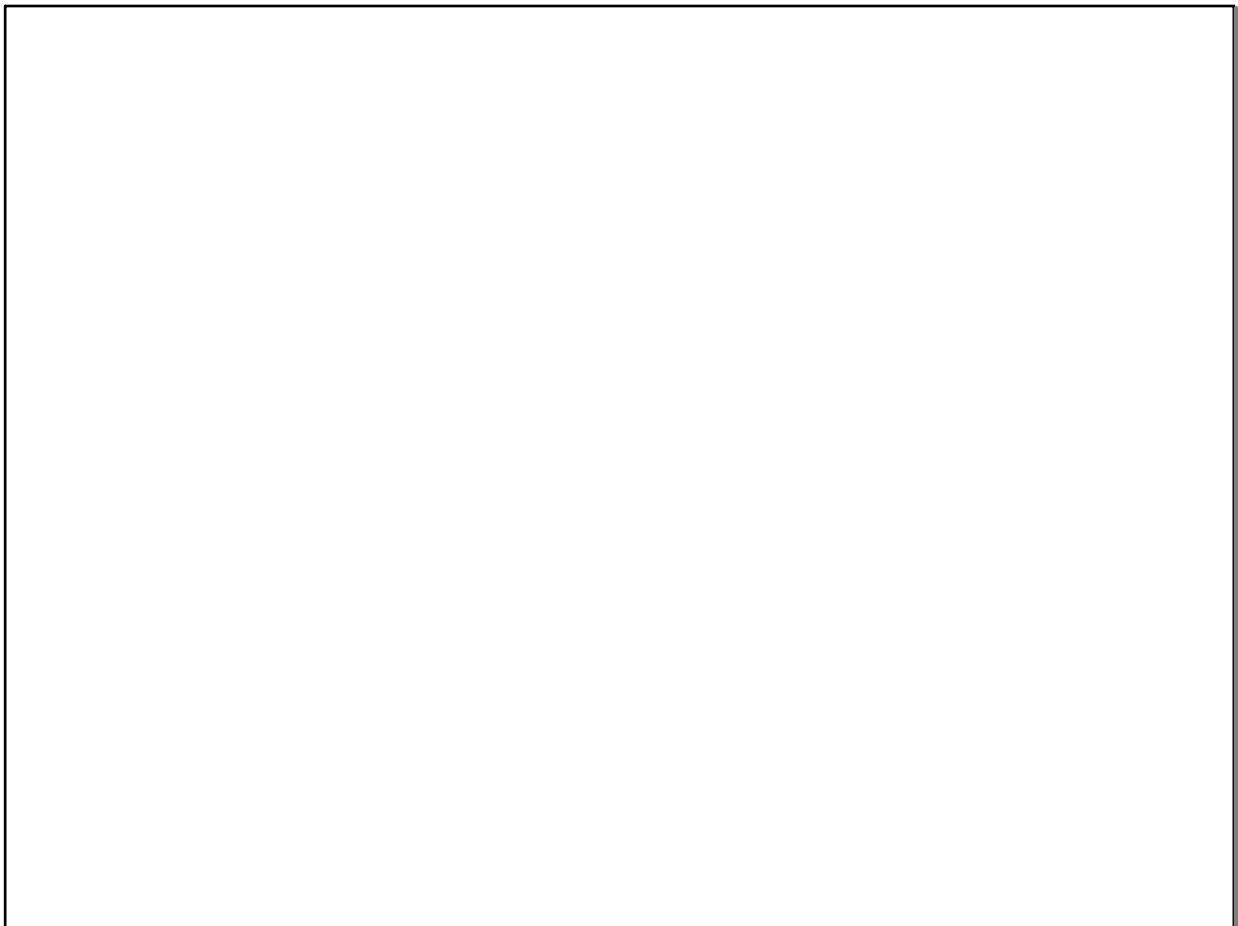
$$x^2 + 2x + 2x + 4$$

$$x^2 + 4x + 4$$

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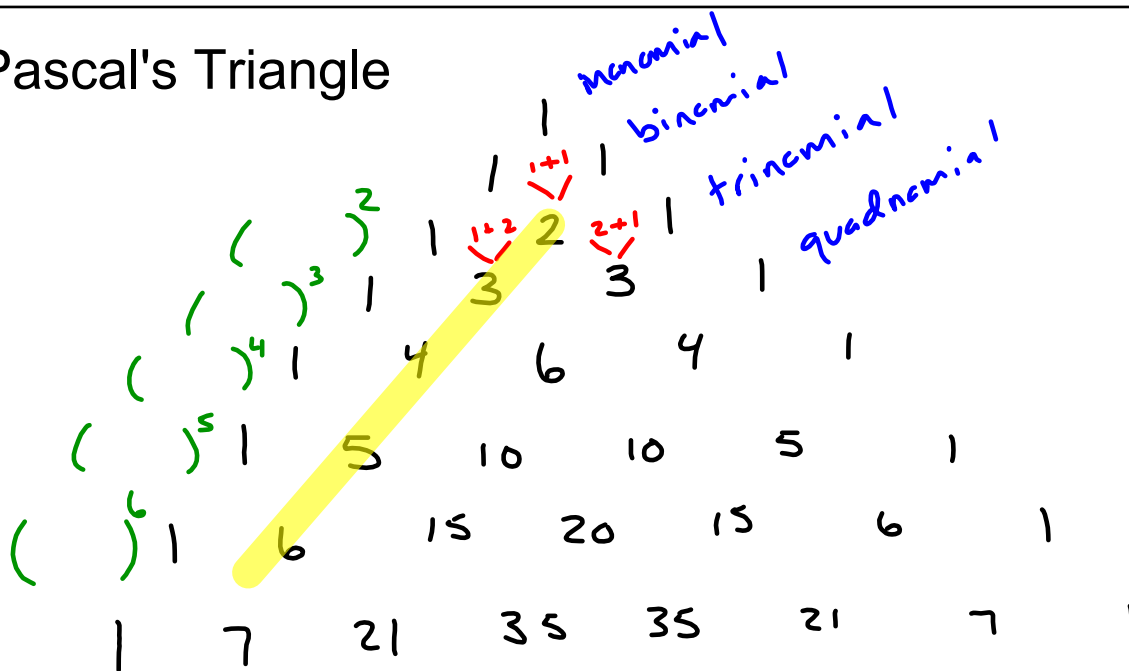


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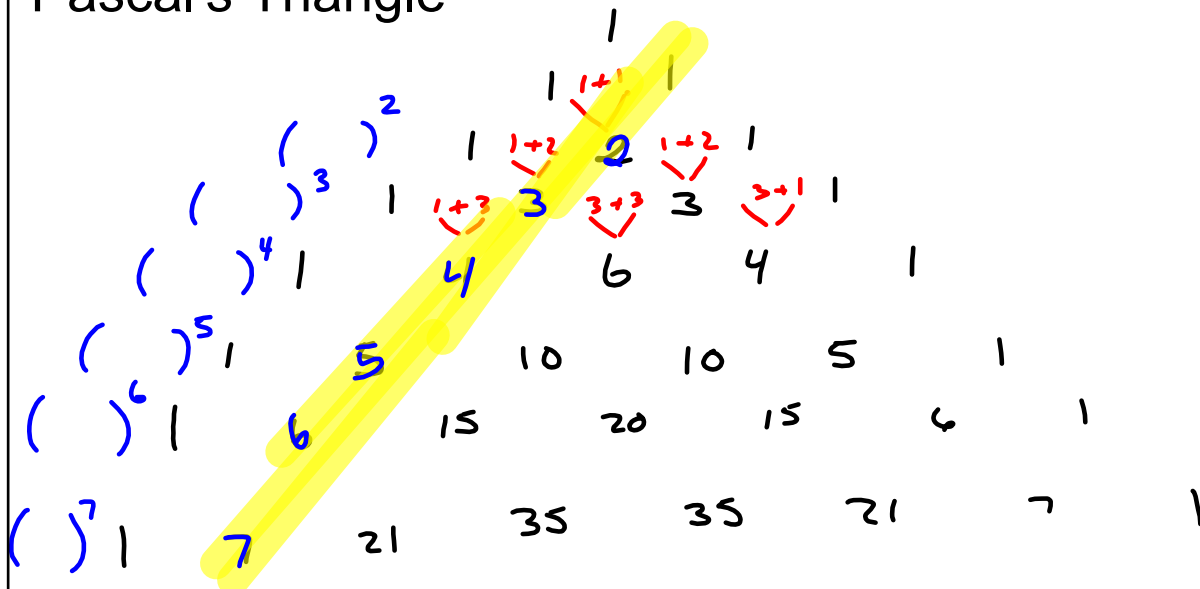
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Pascal's Triangle



Aug 7-7:04 PM

Pascal's Triangle



Aug 7-7:04 PM

$$(x+y)^4$$

$$(x+y)(x+y)(x+y)(x+y)$$

$$x^2 + xy + xy + y^2$$

$$(x^2 + 2xy + y^2)(x^2 + 2xy + y^2)$$

$$x^4 + 2x^3y + x^2y^2 + 2x^3y + 4x^2y^2 + 2xy^3 + x^2y^2 + 2xy^3 + y^4$$

$$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

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PASCAL Δ $(x+y)^4$

Row \Rightarrow 1 4 6 4 1

$1 \binom{4}{x} \binom{0}{y}$	$4 \binom{3}{x} \binom{1}{y}$	$6 \binom{2}{x} \binom{2}{y}$	$4 \binom{1}{x} \binom{3}{y}$	$1 \binom{0}{x} \binom{4}{y}$
$1 \cdot x^4 \cdot 1$	$4 \cdot x^3 \cdot y$	$6x^2y^2$	$4 \cdot x \cdot y^3$	$1 \cdot 1 \cdot y^4$
x^4	$+ 4x^3y$	$+ 6x^2y^2$	$+ 4xy^3$	$+ y^4$

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$$(x+2)^4$$

$1(x)^4(2)^0$ $1 \cdot x^4 \cdot 1$ x^4	$4(x)^3(2)^1$ $4 \cdot x^3 \cdot 2$ $+ 8x^3$	$6(x)^2(2)^2$ $6 \cdot x^2 \cdot 4$ $+ 24x^2$	$4(x)^1(2)^3$ $4 \cdot x \cdot 8$ $+ 32x$	$1(x)^0(2)^4$ $1 \cdot 1 \cdot 16$ $+ 16$
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Aug 8-12:52 PM

$$(-2x - 3)^5$$

$1(-2x)^5(-3)^0$ 5 10 10 5 1

more practice
 3-3

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Pascals Δ

$$\begin{array}{cccccc}
 & & & & 1 & & & & \\
 & & & & 1 & & 1 & & \\
 & & & 1 & & 2 & & 1 & \\
 & & 1 & & 3 & & 3 & & 1 \\
 1 & & 4 & & 6 & & 4 & & 1
 \end{array}$$

$(x+y)^4$

$(x+y)(x+y)(x+y)(x+y)$

$x^2+xy+xy+y^2$

$(x^2+2xy+y^2)(x^2+2xy+y^2)$

$x^4 + 2x^3y + x^2y^2 + 2x^2y + yx^2y^2 + 2xy^3$

$x^2y^2 + 2xy^3 + y^4$

$1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

Aug 8-8:29 AM

$(x+y)^4$

$1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

$(x+y)^4$

$1x^4 \quad 4x^3y \quad 6x^2y^2 \quad 4xy^3 \quad 1y^4$

Process

PASCAL Δ coefficients

Label order of # & variables

Evaluate

Aug 8-8:40 AM

$(x-3)^3$

			1		
		1		1	
	1	3	3	1	
1	4	6	4	1	
1	5	10	10	5	1

$1(x)^3(-3)^0$	$3(x)^2(-3)^1$	$3(x)^1(-3)^2$	$1(x)^0(-3)^3$
$1 \cdot x^3 \cdot 1$	$3 \cdot x^2 \cdot (-3)$	$3 \cdot x \cdot 9$	$1 \cdot 1 \cdot -27$
x^3	$-9x^2$	$+27x$	-27

$x^3 - 9x^2 + 27x - 27$

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$(x-1)^5$

$1(x)^5(-1)^0$	$5(x)^4(-1)^1$	$10(x)^3(-1)^2$	$10(x)^2(-1)^3$	$5(x)^1(-1)^4$	$1(x)^0(-1)^5$
$1 \cdot x^5 \cdot 1$	$5 \cdot x^4 \cdot -1$	$10 \cdot x^3 \cdot 1$	$10 \cdot x^2 \cdot -1$	$5 \cdot x \cdot 1$	$1 \cdot 1 \cdot -1$
x^5	$-5x^4$	$+10x^3$	$-10x^2$	$+5x$	-1

$(-3)^{2 \cdot 4 \cdot 6 \cdot 8} = +$
 $(-3)^{1 \cdot 3 \cdot 5 \cdot 7} = -$

Aug 11-8:23 AM

Division of polynomials

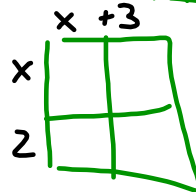
expand the polynomial

$$(x+3)(x+2)$$

$$x^2 + 2x + 3x + 6$$

$$x^2 + 5x + 6$$

FOIL
Distribute
make the



what does $(x+3)$ and $(x+2)$
describe about the polynomial $x^2 + 5x + 6$?
they are the factors.

Aug 11-8:32 AM

* Possible Roots Solve for x
x-int.
FACTOR
 ← this also means

$$\underset{q}{4}x^4 - 21x^3 + 18x^2 + 19x - \underset{p}{6} = 0$$

think $(x)(x) = 0$

Rational Root Theorem:
 $\pm \frac{p}{q}$ will list all possible roots (all $x = _$)
 $p \Rightarrow$ factor of the constant
 $q \Rightarrow$ the factor of the leading coefficient

$$\underset{\text{FACTOR}}{4}x^4 - 21x^3 + 18x^2 + 19x - \underset{\text{FACTOR}}{6} = 0$$

$p \Rightarrow (-6) : \pm 1, \pm 2, \pm 3, \pm 6$
 1-6
 2-3
~~3-2~~

$q \Rightarrow (4) : \pm 1, \pm 2, \pm 4$
 1-4
 2-2
~~4-1~~

$\frac{p}{q} = \frac{\textcircled{1}, \textcircled{2}, \textcircled{3}, \textcircled{6}}{1, 2, 4}$ $\frac{6}{4} = \frac{3}{2}$

$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 3\frac{1}{4}, \pm 6$

Aug 11-8:39 AM

$(x-1)^3$
 $(-1)^{2 \dots 4 \dots 6 \dots 8} = +$
 $(-1)^{3 \dots 5 \dots 7 \dots 9} = -$

$(x-1)$

$(-1)(-1)$

$1(x)(-1)^0$	$3(x)(-1)^1$	$3(x)(-1)^2$	$1(x)(-1)^3$
$1 \cdot x^3 \cdot 1$	$3 \cdot x^2 \cdot -1$	$3 \cdot x \cdot 1$	$1 \cdot 1 \cdot -1$
x^3	$-3x^2$	$+3x$	-1

$x^3 - 3x^2 + 3x - 1$

Aug 11-9:12 AM

$(x-2)^4$

$(x-2)$

$1(x)(-2)^0$	$4(x)(-2)^1$	$6(x)(-2)^2$	$4(x)(-2)^3$	$1(x)(-2)^4$
$1 \cdot x^4 \cdot 1$	$4 \cdot x^3 \cdot -2$	$6 \cdot x^2 \cdot 4$	$4 \cdot x \cdot -8$	$1 \cdot 1 \cdot 16$
x^4	$-8x^3$	$+24x^2$	$-32x$	$+16$

$x^4 - 8x^3 + 24x^2 - 32x + 16$

Aug 11-9:28 AM

polynomials

highest degree $x^2 + 3x^2 + x - 5 = 0$

highest degree (largest exponent) $x^5 + 2x^2 - 4 = 0$

the number of possible real roots

tells us that we have this many parenthesis

points

$-3 -2$

these are roots

x -intercepts $(x, 0)$

solve for x ; $x =$

zeros

$x = -3 \quad x = -2$

$(x+3)(x+2) = 0$

$x^2 + 2x + 3x + 6 = 0$

$x^2 + 5x + 6 = 0$

what does $(x+3)$ and $(x+2)$ represent in the polynomial $x^2 + 5x + 6$?

FACTORS of $x^2 + 5x + 6$

FACTOR $x^2 + 5x + 6$

$(x+3)(x+2) = 0$

Aug 11-9:35 AM

$(x^2 + 5x + 6) \div (x + 2)$

FACTORS $x + 3$

$x + 2 \overline{) x^2 + 5x + 6}$

$-(x^2 + 2x)$

$0 + 3x + 6$

$-(3x + 6)$

$0 + 0 = \text{no remainder}$

$(x+2)(x+3)$

Aug 11-9:47 AM

Rational Root Theorem

$\pm P/Q$ $p \Rightarrow$ FACTOR of the constant
 $q \Rightarrow$ Factors of the leading coefficient.

$4x^5 + 2x^4 + 3x^3 - 4x^2 + x + 6 = 0$

q (under 4) p (under 6)

coefficient w/ largest exponent

Factors (6) $\Rightarrow \pm 1, \pm 2, \pm 3, \pm 6$

Factor (4) $\Rightarrow \pm 1, \pm 2, \pm 4$

Factor of (6) $P/Q = \pm \frac{1, 2, 3, 6}{1, 2, 4}$

1.6
2.3
~~3.2~~

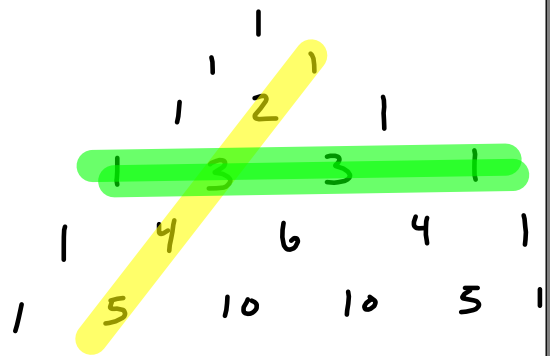
$P/Q = \pm 1, \pm 1/2, \pm 1/4, \pm 2, \pm 3, \pm 3/2, \pm 3/4, \pm 6$

Factors of 4

1.4
2.2

Aug 11-9:50 AM

$(x-2)^3$



$x-2$			
$1(x-2)^0$	$3(x-2)^1$	$3(x-2)^2$	$1(x-2)^3$
$1 \cdot x^0$	$3 \cdot x^1 \cdot -2$	$3 \cdot x^2 \cdot 4$	$1 \cdot 1 \cdot -8$
x^3	$-6x^2$	$+12x$	-8

$x^3 - 6x^2 + 12x - 8$

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$$(x-3)^4$$

$1(x-3)^0$	$4(x-3)^1$	$6(x-3)^2$	$4(x-3)^3$	$1(x-3)^4$
x^4	$4 \cdot x^3 \cdot -3$	$6x^2 \cdot 9$	$4x \cdot -27$	$1 \cdot 1 \cdot 81$
	$-12x^3$	$54x^2$	$-108x$	81

$$x^4 - 12x^3 + 54x^2 - 108x + 81$$

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polynomial

$(x+3)(x+2)$

$x^2 + 3x + 2x + 6$

$x^2 + 5x + 6$

$x = -3$ $x = -2$

-3 -2

What are these points?

Zeros
x-intercepts $(-3, 0)$ and $(-2, 0)$
Solve for x $x = -3$ $x = -2$
roots

$() ()$

FACTORS

What does $(x+3)$ and $(x+2)$ represent in the polynomial $x^2 + 5x + 6$?

$() = 0$

Aug 11-12:32 PM

$$8 \div 4 = 2$$

FACTORS

$$\textcircled{4(2)} = 8$$

$$18 \div 3 = 6$$
$$3(6) = 18$$

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$$\begin{array}{r} (x+2) \overline{) x^2 + 5x + 6} \\ \underline{-(x^2 + 2x)} \\ 0 + 3x + 6 \\ \underline{-(3x + 6)} \\ 0 + 0 \text{ No remainder} \end{array}$$

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$$8 \div \boxed{4} = \boxed{2}$$

$$(4)(2) = 8$$

these are FACTORS of 8

$$18 \div 3 = 6$$

$$3(6) = 18$$

Aug 11-9:44 AM

possible real roots

$$x^2 + 5x + 6 = 0$$

$$(x+3)(x+2) = 0$$

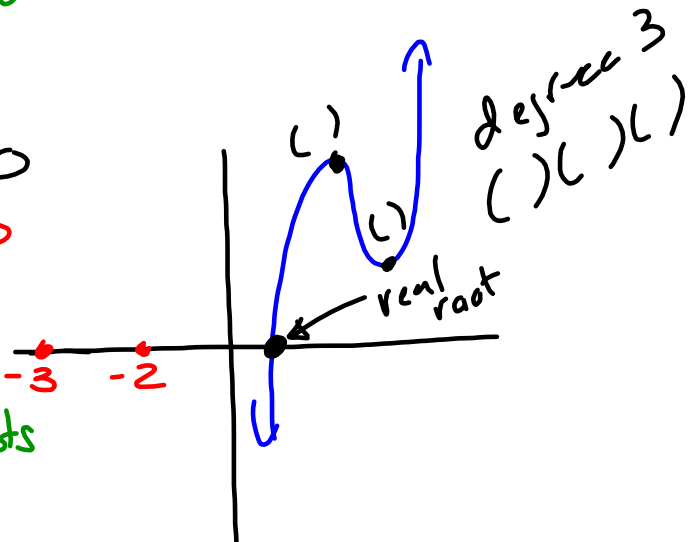
$$x+3=0$$

$$x = -3$$

$$x+2=0$$

$$x = -2$$

x-intercepts



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Rational Root Theorem

$\pm \frac{p}{q}$ $q =$ the factors of the leading coefficient $p =$ the factors of the constant

$$4x^5 - 3x^4 + 2x^3 + x^2 - 6x + 10 = 0$$

1.4

2.2

1.10

2.5

~~5.2~~

$$\pm \frac{p}{q} = \frac{\pm 1, 2, 5, 10}{\pm 1, 2, 4}$$

$$\frac{2}{4} = \frac{1}{2}$$

$$\frac{10}{4} = \frac{5}{2}$$

$$\pm \frac{p}{q} = \pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2, \pm 5, \pm \frac{5}{2}, \pm \frac{5}{4}, \pm 10,$$

x =

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36

1. 36
2. 18
3. 12
4. 9
6. 6

144

1. 144
2. 72
3. 48
4. 36
6. 24
8. 18
9. 16
12. 12

Aug 11-1:01 PM

$$(4)x^4 - 21x^3 + 18x^2 + 19x - 6 = 0$$

$\pm \frac{p}{q} \Rightarrow \pm \frac{1, 2, 3, 6}{1, 2, 4} = \pm 1, \frac{1}{2}, \frac{1}{4}, \dots$

$(x-1)=0$

Synthetic $x=1$

4	-21	18	19	-6	
↓	4	-17	1	20	14
4	-17	1	20	14	

Remainder

Aug 11-1:03 PM

Basic:

$8 \div 4 = 2$

$(4)(2) = 8$
FACTORS

$(x+2)(x+3) = x^2 + 5x + 6$
FACTORS

$18 \div 3 = 6$

$3(6) = 18$

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How to use Pascal's Triangle

Aug 7-7:04 PM

Practice problems

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