## Geometric Sequence



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A **sequence** is an ordered list of numbers.
The **sum** of the terms of a sequence is called a **series.**

If a sequence of values follows a pattern of **multiplying a fixed amount** (not zero) times each term to arrive at the following term, it is referred to as a **geometric sequence.** The number multiplied each time is constant (always the same).

The fixed amount multiplied is called the **common ratio, r,** referring to the fact that the ratio (fraction) of the second term to the first term yields this common multiple. To find the common ratio, divide the second term by the first term.

Notice the **non-linear nature** of the scatter plot of the terms of a geometric sequence. The domain consists of the counting numbers 1, 2, 3, 4, ... and the range consists of the terms of the sequence. While the $x$ value increases by a constant value of one, the $y$ value increases by multiples of two (for this graph).

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## Geometric Sequences ✗ MULTIPLY

**Examples:** $\dfrac{10}{5} = 2$    $\dfrac{20}{10} = 2$    $\dfrac{40}{20} = 2$

| Geometric Sequence | | Common Ratio, $r$ |
|---|---|---|
| $5, 10, 20, 40, \ldots$  $r = 2$  ×2 | $r = 2$ | **multiply** each term by 2 to arrive at the next term or...**divide** $a_2$ by $a_1$ to find the common ratio, 2. |
| $-11, 22, -44, 88, \ldots$  $r = -2$ | $r = -2$ | **multiply** each term by -2 to arrive at the next term .or...**divide** $a_2$ by $a_1$ to find the common ratio, -2. |
| $4, \dfrac{8}{3}, \dfrac{16}{9}, \dfrac{32}{27}, \dfrac{64}{81}, \ldots$  $r =$ | $r = \dfrac{2}{3}$ | **multiply** each term by 2/3 to arrive at the next term or...**divide** $a_2$ by $a_1$ to find the common ratio, 2/3. |

$\dfrac{22}{-11} = -2$    $\dfrac{-44}{22} = -2$    $\dfrac{88}{-44} = -2$

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$4, \dfrac{8}{3}, \dfrac{16}{9}, \dfrac{32}{27}, \dfrac{64}{81}, \ldots$    $\boxed{r = \dfrac{2}{3}}$

$\dfrac{\frac{8}{3}}{\frac{4}{1}}$    $\dfrac{8}{3} \cdot \dfrac{1}{4} = \dfrac{8}{12} = \dfrac{2}{3}$

$\dfrac{16/9}{8/3}$    $\dfrac{16}{9} \cdot \dfrac{3}{8} = \dfrac{48}{72} = \dfrac{2}{3}$

$\dfrac{32/27}{16/9}$    $\dfrac{32}{27} \cdot \dfrac{9}{16} = \dfrac{288}{432} = \dfrac{2}{3}$

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**2**

**Formulas used with geometric sequences and geometric series:**

**geometric sequences**

To **find any term** of a geometric sequence:

$$a_n = a_1 \cdot r^{n-1}$$

where $a_1$ is the first term of the sequence, $r$ is the common ratio, $n$ is the number of the term to find.

*Note:* $a_1$ is often simply referred to as $a$.

**geometric series:**

To find the **sum of a certain number of terms** of a geometric sequence:

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

where $S_n$ is the sum of $n$ terms ($n^{th}$ **partial sum**), $a_1$ is the first term, $r$ is the common ration.

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To find the **sum of a certain number of terms** of a geometric sequence:

$$S_{n=30} = \frac{\overset{.01}{a_1}(1 - r^n)}{1 - r}$$

where $S_n$ is the sum of $n$ terms ($n^{th}$ **partial sum**), $a_1$ is the first term, $r$ is the common ration.

$$S_{30} = \frac{.01\left(1 - 2^{30}\right)}{1 - 2}$$

$$10{,}737{,}418.23$$

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| Question | Answer |
|---|---|
| 1. Find the common ratio for the sequence<br><br>$6,\ -3,\ \dfrac{3}{2},\ -\dfrac{3}{4},\ ...$ | 1. The common ratio, $r$, can be found by dividing the second term by the first term, which in this problem yields -1/2. Checking shows that multiplying each entry by -1/2 yields the next entry. |
| 2. Find the common ratio for the sequence given by the formula<br><br>$a_n = 5(3)^{n-1}$<br><br>$\overline{\quad}$ , $\overline{\quad}$ , $\overline{\quad}$ , $\overline{\quad}$ , $\overline{\quad}$<br>n=1   n=2   n=3   n=4   n=5 | 2. The formula indicates that 3 is the common ratio by its position in the formula. A listing of the terms will also show what is happening in the sequence (start with $n = 1$).<br><br>$\qquad$ 5, 15, 45, 135, ...<br>The list also shows the common ratio to be 3. |
| 3. Find the 7$^{th}$ term of the sequence<br><br>$\underline{2},\ \underline{6},\ \underline{18},\ \underline{54},\ ...,\ \underline{\quad},\ \underline{\quad},\ \underline{\quad}$ | 3. $n = 7$; $a_1 = 2$, $r = 3$<br><br>$a_n = a_1 \cdot r^{n-1}$<br>$a_7 = 2 \cdot 3^{7-1} = 1458$<br>The seventh term is 1458. |
| 4. Find the 11$^{th}$ term of the sequence<br><br>$1, -\dfrac{1}{2}, \dfrac{1}{4}, -\dfrac{1}{8}, ...$ | 4. $n = 11$; $a_1 = 1$, $r = -1/2$<br><br>$a_{11} = 1 \cdot \left(-\dfrac{1}{2}\right)^{11-1} = \dfrac{1}{1024}$ |

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**4.** Find the 11$^{th}$ term of the sequence

$1, \dfrac{1}{2}, \dfrac{1}{4}, -\dfrac{1}{8}, ...$

n=1   n=2   n=3   ...

$\dfrac{-\frac{1}{2}}{1} = -\dfrac{1}{2}$

$\boxed{\dfrac{1}{1024}}$   $n=11$

$a_{n=11} = a_1 \left(-\tfrac{1}{2}\right)^{n-1}$

$a_{11} = 1\left(-\tfrac{1}{2}\right)^{11-1}$   $\boxed{.000977}$

$= 1\left(-\tfrac{1}{2}\right)^{10} = 9.77 \times 10^{-4}$

**1.** Find the common ratio for the sequence

$$\frac{-\frac{3}{4}}{\frac{3}{2}} \quad \frac{-3}{4} \cdot \frac{2}{3} \qquad 6, \ -3, \ \frac{3}{2}, \ -\frac{3}{4}, \ \dots \qquad \frac{-3}{6} \boxed{\frac{-1}{2}} \qquad \frac{\frac{3}{2}}{-3} \quad \frac{3}{2} \cdot \frac{-1}{3} = \frac{-3}{6}$$

$$-6/12 \qquad \qquad r = \checkmark \quad r = \checkmark \qquad \qquad \boxed{= \frac{-1}{2}}$$

**2.** Find the common ratio for the sequence given by the formula

$$a_n = (5)(3)^{n-1}$$

$$\underset{n=1}{\underline{5}}, \ \underset{n=2}{\underline{15}}, \ \underset{n=3}{\underline{45}}, \ \underset{n=4}{\underline{135}}, \ \underset{n=5}{\underline{405}}$$

$$a_1 \qquad a_2 \qquad a_3 \qquad a_4 \qquad a_5$$

$$r = 3 \qquad r = 3 \qquad r = 3 \qquad r = 3$$

$$15/5 = 3 \qquad 45/15 \qquad 135/45 \qquad 405/135$$

$$a_n = a_1 (r)^{n-1}$$

$$\underset{1st \ term}{\uparrow} \qquad \underset{common \ ratio}{\curvearrowleft}$$

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**4.** Find the 11$^{th}$ term of the sequence

$$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \dots$$

$$r = \frac{-\frac{1}{8}}{1} = \boxed{\frac{-1}{2}} \qquad \frac{1}{}\left(\frac{-1}{2}\right)^{n-1}$$

$$1\left(-\frac{1}{2}\right)^{10} = \frac{1}{1024}$$

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5

$a_1 = .01$

Sequence ⇒ pattern

$.01(2)^{n-1}$

series ⇒ add the patteren

$.01(2)^{29}$

5, 10, 20, 40

Series of the 1ˢᵗ 4 terms

5 + 10 + 20 + 40 = 75

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**Question**

1. Find the common ratio for the sequence

$$6, \quad -3, \quad \frac{3}{2}, \quad -\frac{3}{4}, \quad ...$$

$\dfrac{-3}{6} \left(\dfrac{-1}{2}\right)$    $\dfrac{3/2}{-3}$   $\dfrac{3}{2} \cdot \dfrac{-1}{3} = \dfrac{-3}{6} \left(\dfrac{-1}{2}\right)$

$\dfrac{-3/4}{3/2}$   $\dfrac{-3}{4} \cdot \dfrac{2}{3} = \dfrac{-6}{12}$

2. Find the common ratio for the sequence given by the formula

$$a_n = (5)(3)^{n-1}$$

common ratio

$\left(\dfrac{-1}{2}\right)$

$5, \quad 15, \quad 45, \quad 135, \quad 405$

$n=1 \quad n=2 \quad n=3 \quad n=4 \quad n=5$

$r = 3$

$a_n = a_1 (r)^{n-1}$

3. Find the 7ᵗʰ term of the sequence

$n = 127$    2, 6, 18, 54, ... 162, 486, 1458

$r = 3 \quad r = 3$

Now 127ᵗʰ     $a_{127} = 2(3)^{n-1}$

12ᵗʰ term    7.62 E 60    $2(3)^{126}$

          7.62 × 10⁶⁰

$2(3)^{11} = 354{,}294$

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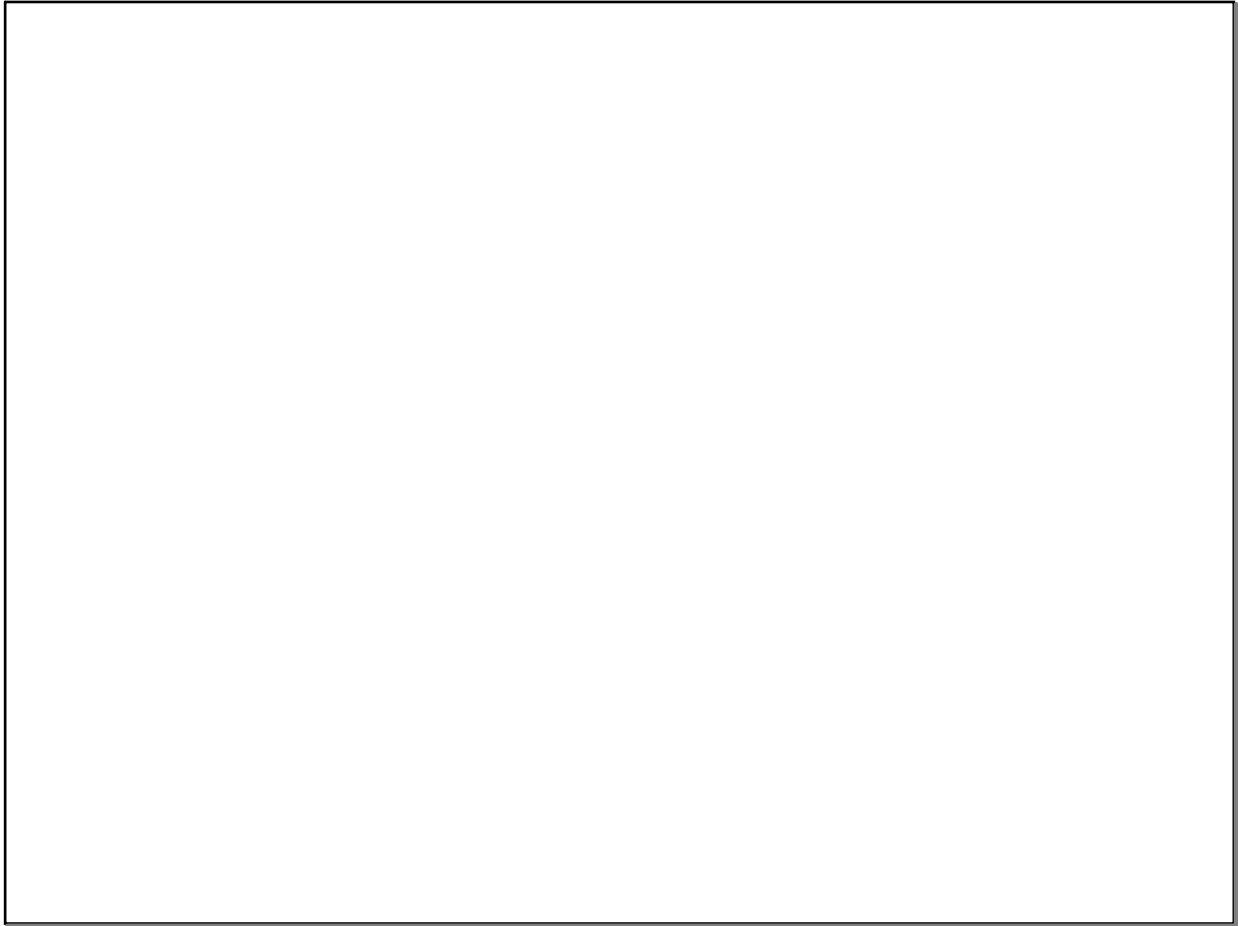| | |
|---|---|
| **5.** Find $a_8$ for the sequence<br><br>$\qquad$ 0.5, 3.5, 24.5, 171.5, ... | **5.** $n = 8;\ a_1 = 0.5,\ r = 7$<br><br>$a_n = a_1 \cdot r^{n-1}$<br><br>$a_8 = 0.5 \cdot 7^{8-1} = 411,771.5$ |
| **6.** Evaluate using a formula:<br><br>$\qquad \displaystyle\sum_{k=1}^{5} 3^k$ | **6.** Examine the summation<br><br>$\displaystyle\sum_{k=1}^{5} 3^k = 3^1 + 3^2 + 3^3 + 3^4 + 3^5$<br><br>This is a geometric series with a common ratio of 3.<br>$n = 5;\ a_1 = 3,\ r = 3$<br><br>$S_5 = \dfrac{3(1-3^5)}{1-3} = \dfrac{-726}{-2} = 363$ |
| **7.** Find the sum of the first 8 terms of the sequence<br><br>$\qquad$ -5, 15, -45, 135, ... | **7.** The word "sum" indicates a need for the sum formula.<br>$n = 8;\ a_1 = -5,\ r = -3$<br>$S_8 = \dfrac{-5(1-(-3)^8)}{1-(-3)}$<br><br>$S_8 = \dfrac{-5(1-6561)}{4} = \dfrac{32800}{4} = 8200$ |

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| | |
|---|---|
| **8.** The third term of a geometric sequence is 3 and the sixth term is 1/9. Find the first term. | **8.** Think of the sequence as "starting with" 3, until you find the common ratio.<br><br>$\underline{\quad}, \underline{\quad}, \boxed{3, \underline{\quad}, \underline{\quad}, \dfrac{1}{9}}$<br><br>*For this modified sequence:* $a_1 = 3,\ a_4 = 1/9,$<br>$n = 4$<br><br>$a_n = a_1 \cdot r^{n-1}$<br><br>$\dfrac{1}{9} = 3 \cdot r^{4-1}$<br><br>$\dfrac{1}{27} = r^3$<br><br>$\dfrac{1}{3} = r$<br><br>Now, work backward multiplying by 3 (or dividing by 1/3) to find the actual first term.<br>$a_1 = 27$ |
| **9.** A ball is dropped from a height of 8 feet. The ball bounces to 80% of its previous height with each bounce. How high (*to the nearest tenth of a foot*) does the ball bounce on the fifth bounce? | **9.** Set up a model drawing for each "bounce".<br>$\qquad$ 6.4, 5.12, $\underline{\quad}, \underline{\quad}, \underline{\quad}$<br>The common ratio is 0.8.<br><br>$a_n = a_1 \cdot r^{n-1}$<br><br>$a_n = 6.4 \cdot (.8)^{5-1} = 2.62144$<br><br>Answer: 2.6 feet |

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**7**

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