

Graphing Polynomials

(x, y)	zeros	Factors
point A is $(-7, 0)$	$x = -7$	$(x + 7)$
point B is $(1, 0)$	$x = 1$	$(x - 1)$
point C is $(5, 0)$	$x = 5$	$(x - 5)$

write the function in factor form

$y = (x + 7)(x - 1)(x - 5)$

expand the polynomial

$y = x^3 + x^2 - 37x + 35$

$(x + 7)(x - 1)(x - 5)$
 $(x^2 - x + 7x - 7)(x - 5)$
 $(x^2 + 6x - 7)(x - 5)$
 $x^3 + 6x^2 - 7x$
 $- 5x^2 - 30x + 35$

 $x^3 + x^2 - 37x + 35$

Aug 30-5:36 PM

- The points on the x-axis are called x-intercepts.
- The degree of the polynomial tells you how many **factors, zeros, solutions, x-intercepts** * imaginary numbers do not touch x-axis.
- Reflections over x-axis, **the y-points change signs**
- Reflections over y-axis, **the x-points change signs**

Aug 30-5:44 PM

How to graph polynomials

1. Factor the polynomial to find x-intercepts
2. Set the factors equal to zero
3. Graph the x-intercepts
4. Now look at sign in front of the high degree exponent to see the starting direction of the graph

Aug 30-5:52 PM

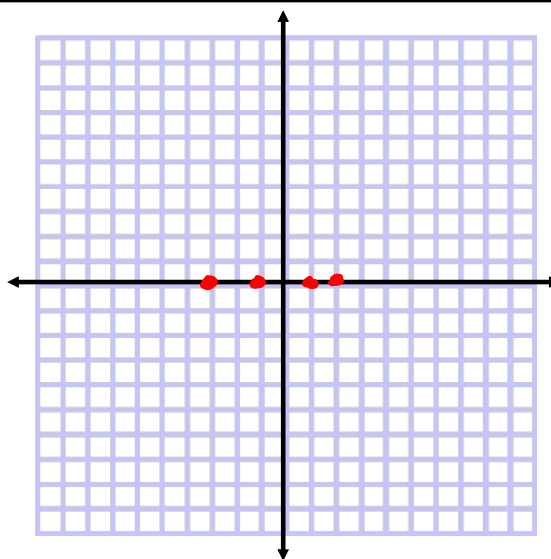
Example Problem

Graph the following function

$$f(x) = x^4 + x^3 - 7x^2 - x + 6$$

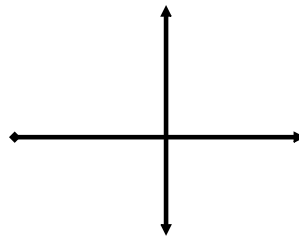
To get the factors

- Find the possible rational roots
- Synthetic division
- Set factors equal to zero
- Plot x-intercepts

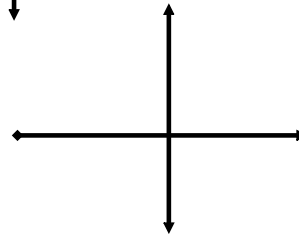


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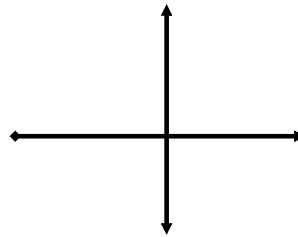
① $x^3 + 3x^2 - 13x - 15$



② $x^3 - 5x^2 + 6x$



③ $x^3 - 7x^2 - 11x + 14$



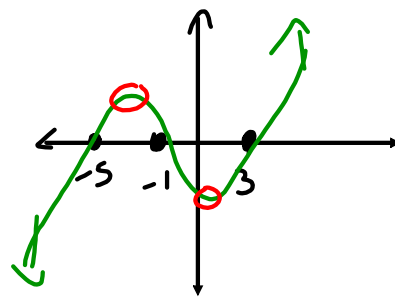
Sep 2-8:48 AM

① $x^3 + 3x^2 - 13x - 15$

$P/q = \pm 1, 3, 5, 15$

$-1 \mid \begin{array}{r} 1 \quad 3 \quad -13 \quad -15 \\ \downarrow \quad -1 \quad -2 \quad 15 \\ \hline 1 \quad 2 \quad -15 \quad 0 \end{array}$

$x^2 + 2x - 15$
 $(x+5)(x-3)$
 $x = -5 \quad x = 3$



$3 \mid \begin{array}{r} 1 \quad 2 \quad -15 \\ \downarrow \quad 3 \quad 15 \\ \hline 1 \quad x \quad 5 \quad 0 \end{array}$
constant
 $x + 5 = 0$
 $x = -5$

Sep 2-8:54 AM

② $x^3 - 5x^2 + 6x + 0$ ~~$(x-6)(x+1)$~~

GCF

$(x)(x^2 - 5x + 6)$

$(x)(x-3)(x-2)$

$x=0$ $x-3=0$ $x-2=0$
 $x=3$ $x=2$

$(x+3)(x-2)$
 $x^2 - 2x + 3x - 6$

$x^2 - 5x + 6$
 $7 = 1 \cdot 6$
 $5 = 2 \cdot 3$
 $-7 = -1 \cdot 6$
 $-5 = -2 \cdot 3$

positive

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③ $x^3 - 7x^2 - 11x + 14$

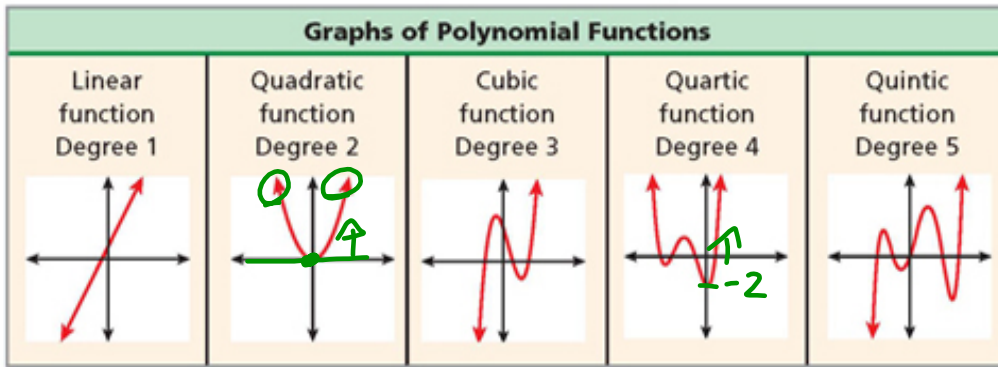
$P/q = \pm 1, 2, 7, 14$

$-2 \mid 1 \quad -7 \quad -11 \quad 14$
 $\quad \downarrow \quad -2 \quad 18 \quad -14$
 $\hline 1 \quad -9 \quad 7 \quad 0$

$x^2 - 9x + 7 = 0$
 $(x \quad)(x \quad) = 0$

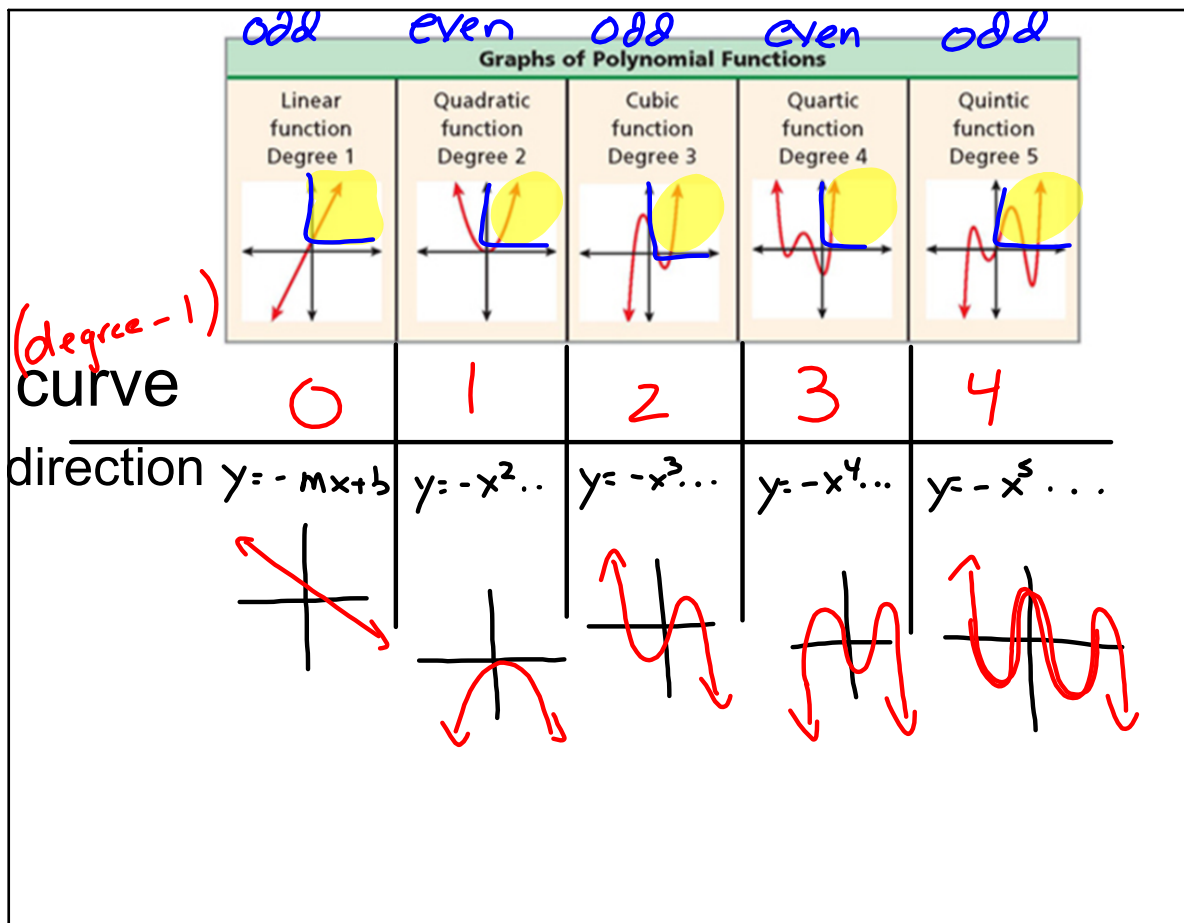
$\frac{9 \pm \sqrt{9^2 - 4(1)(7)}}{2(1)}$
 $\frac{9 \pm \sqrt{81 - 28}}{2}$
 $\frac{9 \pm \sqrt{53}}{2} = \text{decimals}$ 😊

Sep 2-8:54 AM


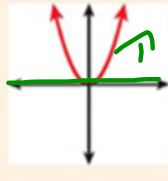
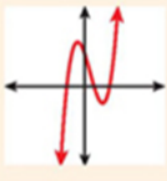
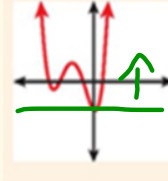
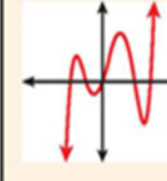


End behavior is a description of the values of the function as x approaches positive infinity ($x \rightarrow +\infty$) or negative infinity ($x \rightarrow -\infty$). The degree and leading coefficient of a polynomial function determine its end behavior. It is helpful when you are graphing a polynomial function to know about the end behavior of the function.

Sep 2-8:53 AM

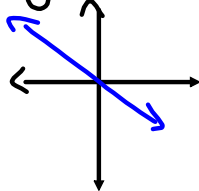
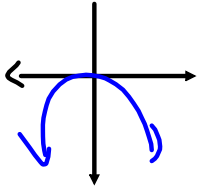
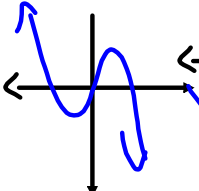
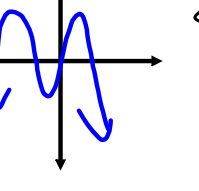
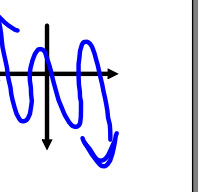


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Linear function Degree 1	Quadratic function Degree 2	Cubic function Degree 3	Quartic function Degree 4	Quintic function Degree 5
				

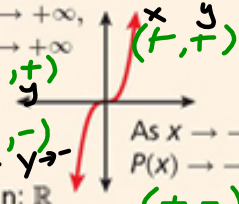
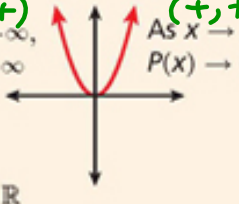
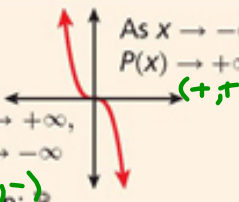
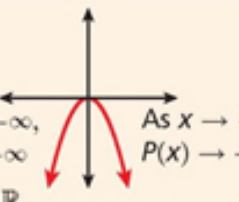
Curves: 0 1 2 3 4

→ (degree-1)

$y = mx + b$	$y = -x^2$	$y = -x^3 \dots$	$y = -x^4 \dots$	$y = -x^5 \dots$
				

Sep 3-8:29 AM

→ Polynomial End Behavior

P(x) has...	Odd Degree	Even Degree
Leading coefficient $a > 0$	As $x \rightarrow +\infty$, $P(x) \rightarrow +\infty$  As $x \rightarrow -\infty$, $P(x) \rightarrow -\infty$ Domain: \mathbb{R} Range: \mathbb{R}	As $x \rightarrow -\infty$, $P(x) \rightarrow +\infty$  As $x \rightarrow +\infty$, $P(x) \rightarrow +\infty$ Domain: \mathbb{R} Range: all values \geq minimum
Leading coefficient $a < 0$	As $x \rightarrow -\infty$, $P(x) \rightarrow +\infty$  As $x \rightarrow +\infty$, $P(x) \rightarrow -\infty$ Domain: \mathbb{R} Range: \mathbb{R}	As $x \rightarrow -\infty$, $P(x) \rightarrow -\infty$  As $x \rightarrow +\infty$, $P(x) \rightarrow -\infty$ Domain: \mathbb{R} Range: all values \leq maximum

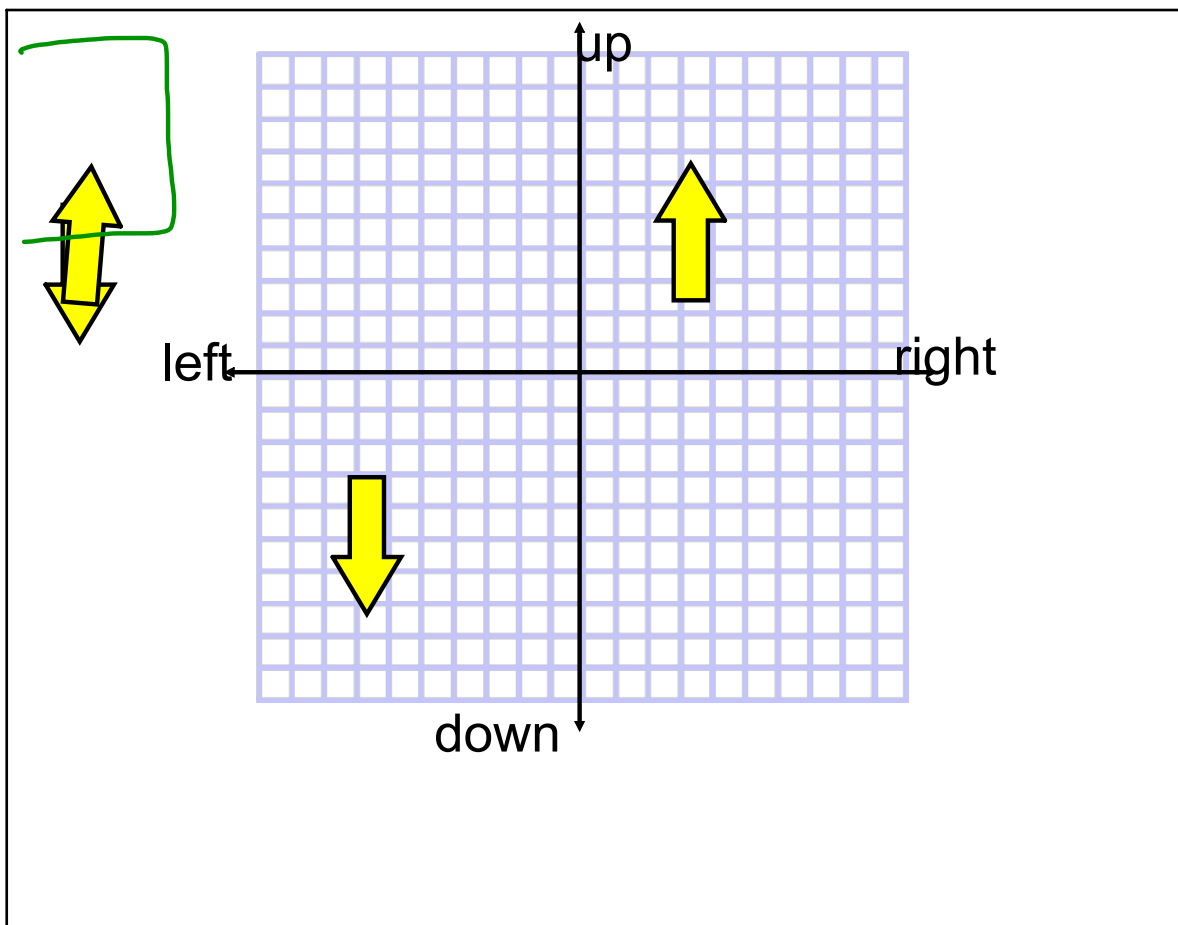
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You need a partner

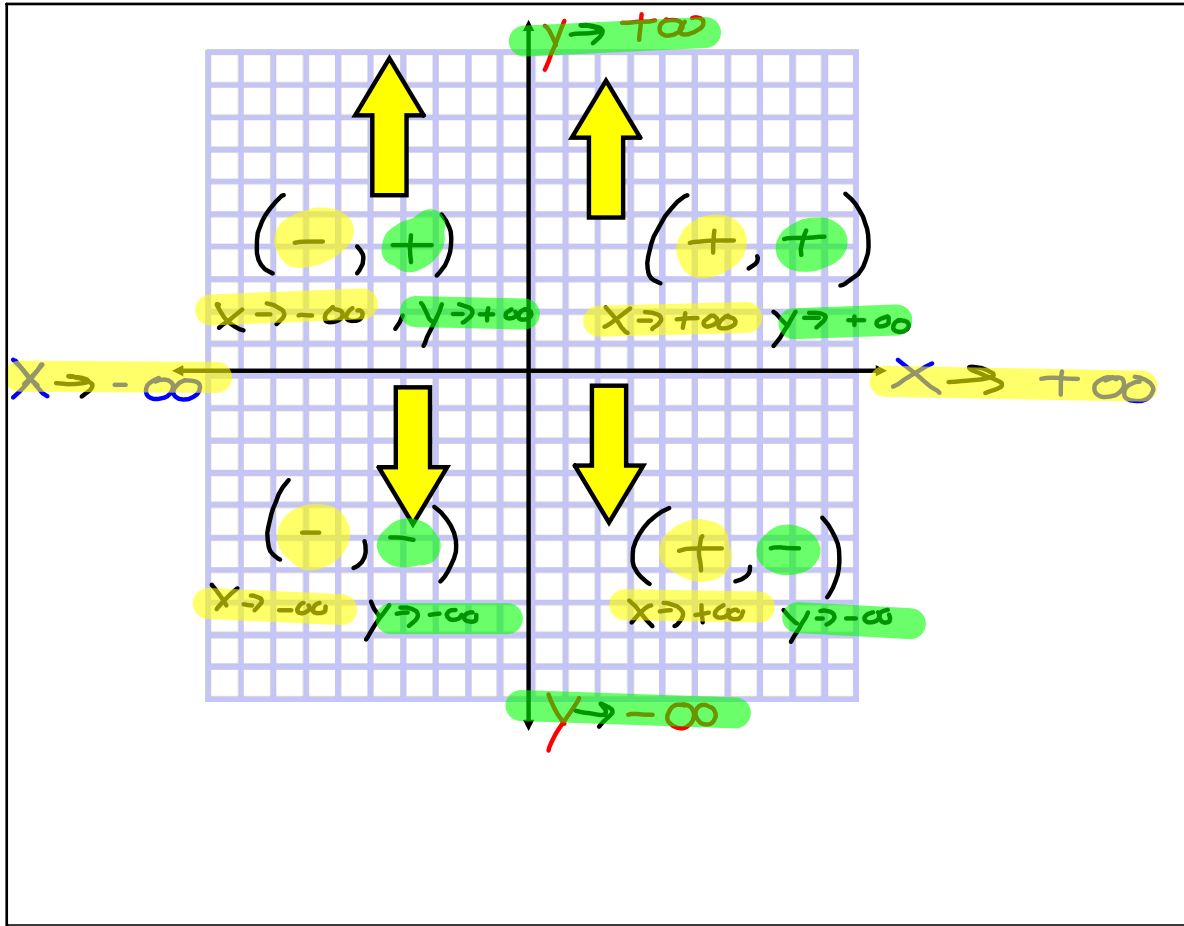
You need a dry erase board

You need a dry erase marker

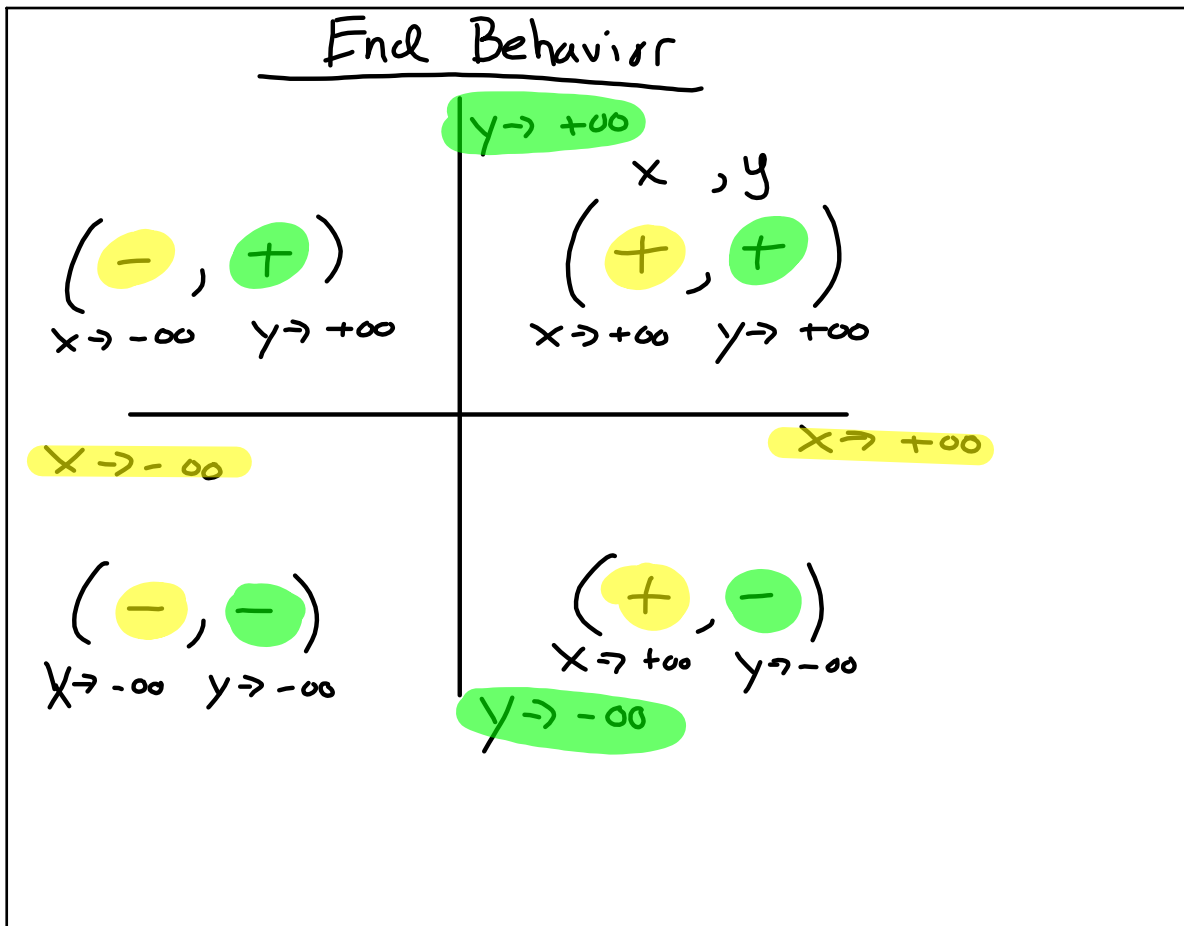
Sep 2-9:10 AM



Sep 2-8:12 AM



Sep 2-8:12 AM



Sep 3-9:45 AM

$x \rightarrow -\infty$ $x \rightarrow +\infty$
 $y \rightarrow -\infty$ $y \rightarrow +\infty$

x -int: $x = -3(x+3)$
 $(x+3)(x-2)(x-5)$ $x = 2(x-2)$
 $x = 5(x-5)$

Sketch the graph.

Sep 3-8:48 AM

$x \rightarrow -\infty$ $x \rightarrow +\infty$
 $y \rightarrow -\infty$ $y \rightarrow +\infty$

x -int: $x = -3$
 $x = 2$
 $x = 5$

Sketch the graph

Sep 3-9:52 AM

① $X^3 - 8x^2 + 11x + 20$

Graph
+
tell end behavior

② $X^4 + X^3 - 13x^2 - x + 12$

$x \rightarrow -\infty$
 $y \rightarrow$

$x \rightarrow +\infty$
 $y \rightarrow$

③ $X^4 - 10x^2 + 9$

Sep 3-9:00 AM

① $X^3 - 8x^2 + 11x + 20$

$\mathcal{P}/\mathcal{Q} = \pm 1, 2, 4, 5, 10, 20$

$x=5$

1	- 8	11	20
↓	5	-15	-20
1	- 3	- 4	0

$x^2 - 3x - 4$
 $(x - 4)(x + 1)$
 $x = 4$ $x = -1$

Sep 4-9:13 AM

② $x^4 + x^3 - 13x^2 - x + 12$

$p/q = \pm 1, 2, 3, 4, 6, 12$

$x = 1$

1	1	-13	-1	12	
↓	1	2	-11	-12	
	1	2	-11	-12	0

$x = -1$

1	2	-11	-12	
↓	-1	-1	+12	
	1	1	-12	0

$x^2 + x - 12$
 $(x + 4)(x - 3)$
 $x = -4$ $x = 3$

Sep 4-9:13 AM

③ $x^4 - 10x^2 + 9$

$p/q = \pm 1, 3, 9$

$x^4 + 0x^3 - 10x^2 + 0x + 9$

$x = 1$

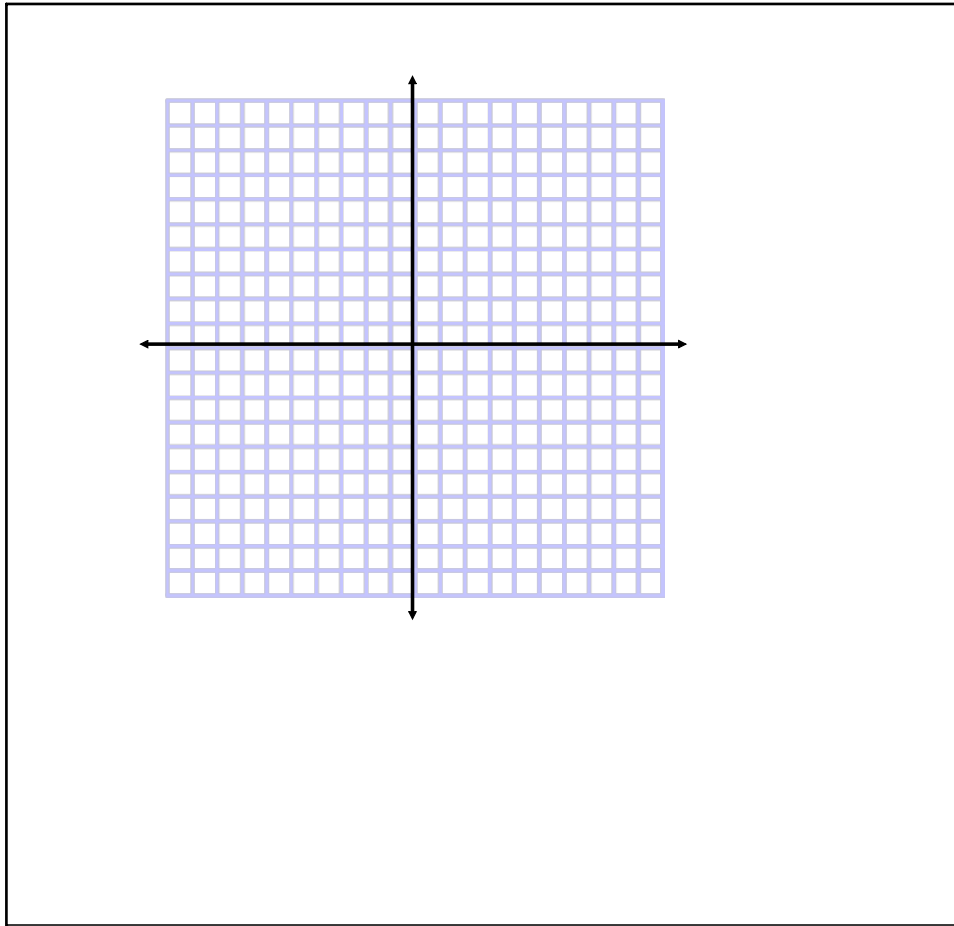
1	0	-10	0	9	
↓	1	1	-9	-9	
	1	1	-9	-9	0

$x = -1$

1	1	-9	-9	
↓	-1	0	9	
	1	0	-9	0

$x^2 - 9 = 0$ $x^2 - 9 = 0$
 $(x - 3)(x + 3)$ $x^2 = 9$
 $x = 3$ $x = -3$ $x = \pm\sqrt{9}$
 $x = \pm 3$

Sep 4-9:13 AM



Sep 4-11:51 AM

① $x^3 - 8x^2 + 11x + 20$

$p/q = \pm 1, 2, 4, 5, 10, 20$

5	1	-8	11	20
	↓	5	-15	-20
	1	-3	-4	0

$x^2 - 3x - 4$
 $(x+1)(x-4)$

$x = -5 \quad x = -1 \quad x = 4$

$y = -x^3 \dots$

Sep 4-8:16 AM

② $x^4 + x^3 - 13x^2 - x + 12$

$P/q = \pm 1, 2, 3, 4, 6, 12$

① \downarrow

1	1	-13	-1	12
\downarrow	1	2	-11	-12
1	2	-11	-12	0

② \downarrow

1	2	-11	-12
\downarrow	-1	-1	+12
1	1	-12	0

$x^2 + x - 12$
 $(x+4)(x-3)$
 $x = -4$ $x = 3$

Sep 4-8:16 AM

$x^4 + 0x^3 - 10x^2 + 0x + 9$

③ $x^4 - 10x^2 + 9$

$P/q = \pm 1, 3, 9$

① \downarrow

1	0	-10	0	9
\downarrow	1	1	-9	-9
1	1	-9	-9	0

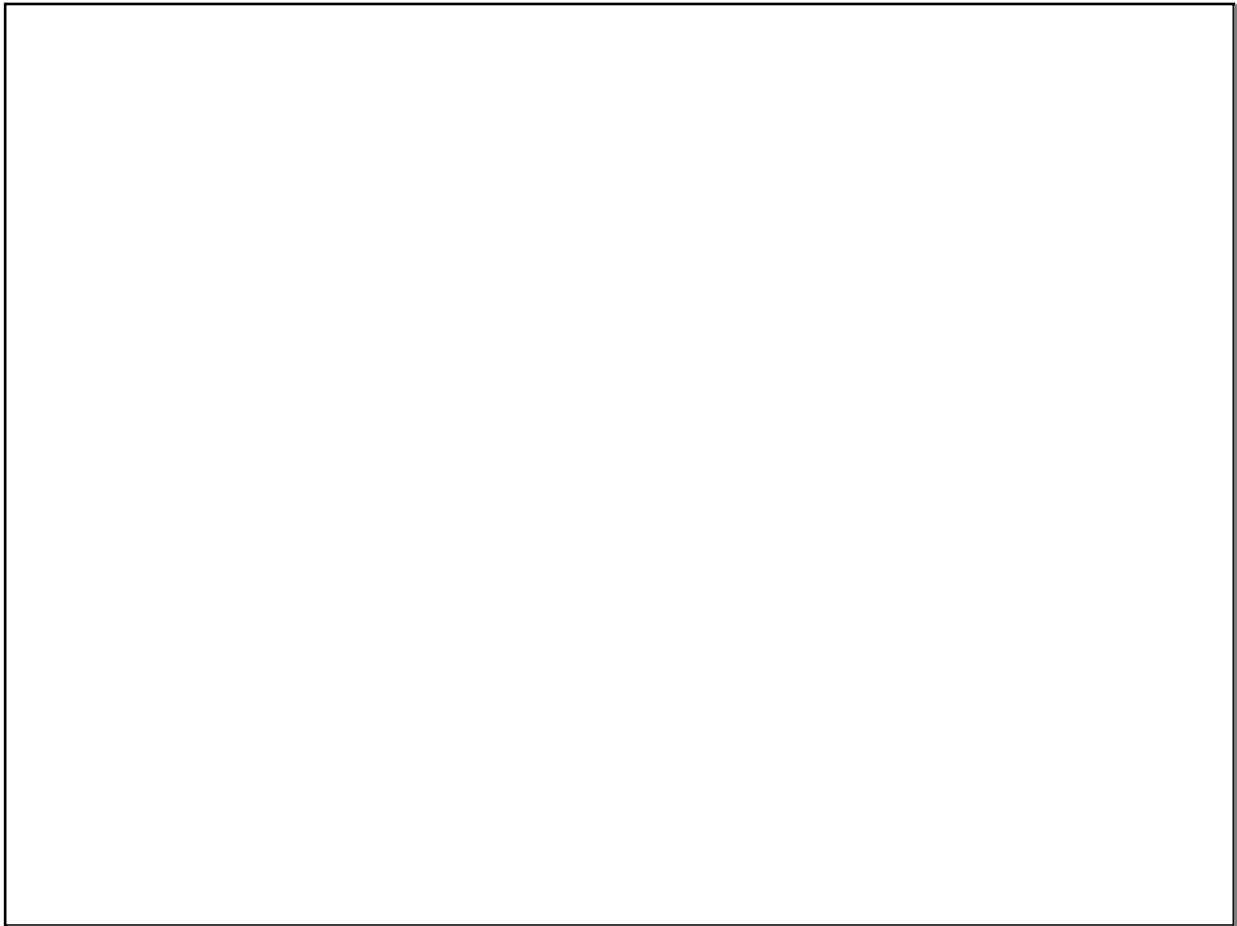
② \downarrow

1	1	-9	-9
\downarrow	-1	0	+9
1	0	-9	0

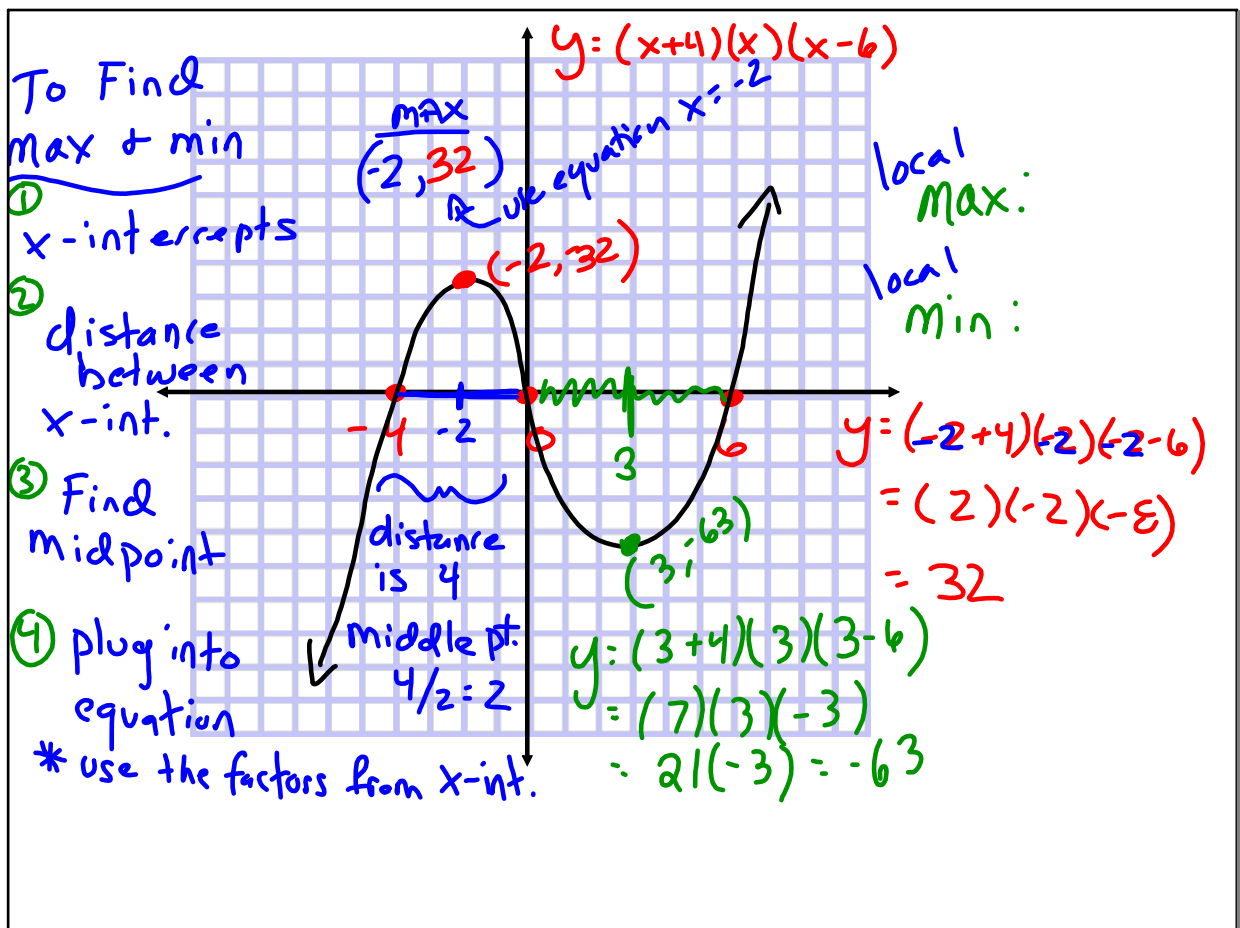
$x^2 - 9$
 $(x+3)(x-3)$
 $x = -3$ $x = 3$

$x^2 - 9 = 0$
 $x^2 = 9$
 $x = \pm\sqrt{9}$
 $x = \pm 3$

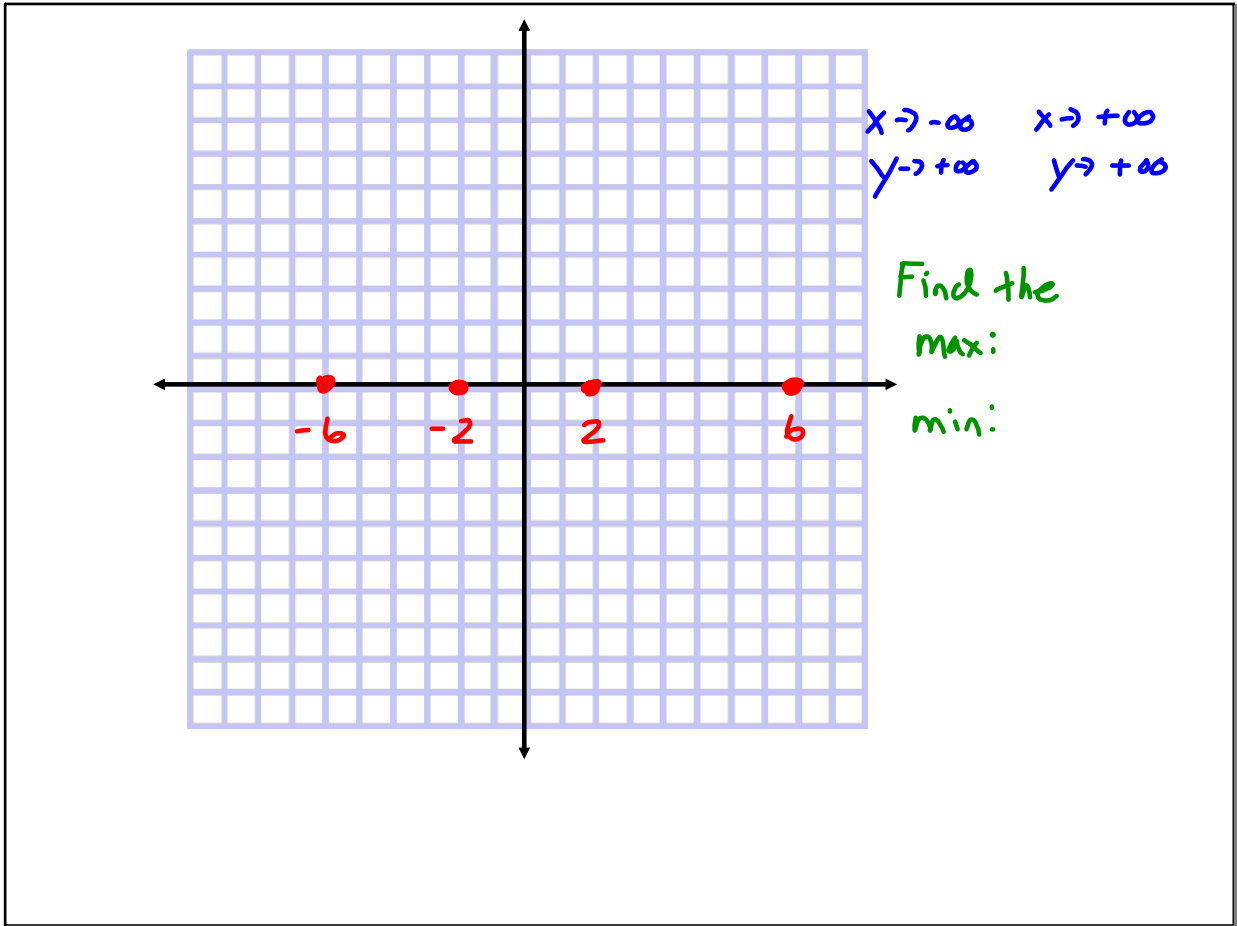
Sep 4-8:16 AM



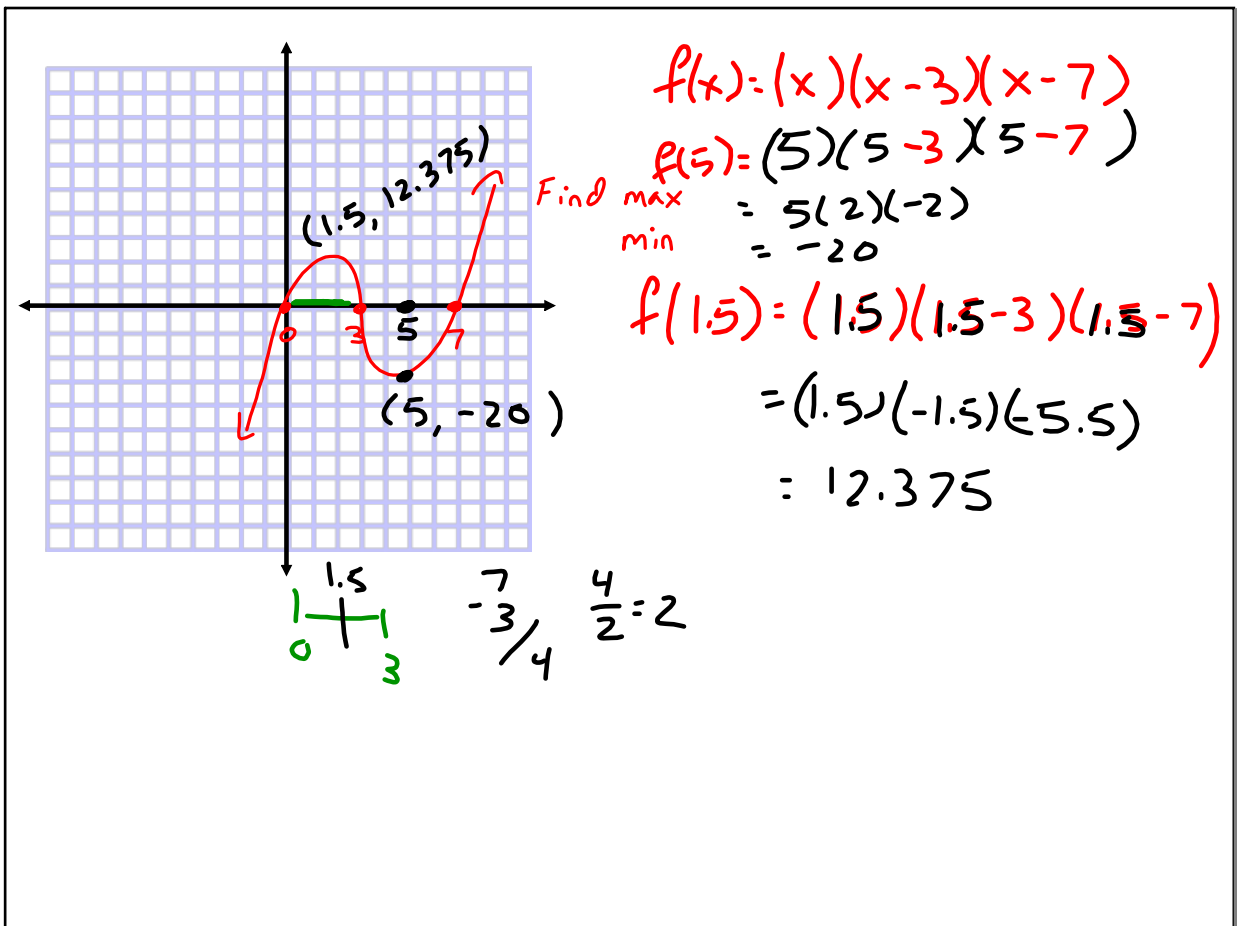
Sep 4-8:10 AM



Sep 2-8:12 AM



Sep 2-8:12 AM



Sep 2-8:12 AM

To Find max + min

- ① x-intercepts
- ② distance between x-int.
- ③ Find midpoint
- ④ plug into equation

* use the factors from x-int.

$x = -4 \quad (x+4)$
 $x = 0 \quad (x)$
 $x = 6 \quad (x-6)$

$f(x) = (x+4)(x)(x-6)$

max: $(-2, y)$ $f(-2) = (-2+4)(-2)(-2-6)$
 $x = -2 \quad y = ?$ $x = -2$
 $y = 32$ $= (2)(-2)(-8)$
 $= (-4)(-8) = 32$

min: $(3, y)$ $f(3) = (3+4)(3)(3-6)$
 $x = 3 \quad y = ?$
 $= (7)(3)(-3)$
 $= 21(-3) = -63$

Sep 2-8:12 AM

$x \rightarrow -\infty \quad (-, +)$
 $x \rightarrow +\infty \quad (+, +)$
 $y \rightarrow +\infty$
 $y \rightarrow +\infty$

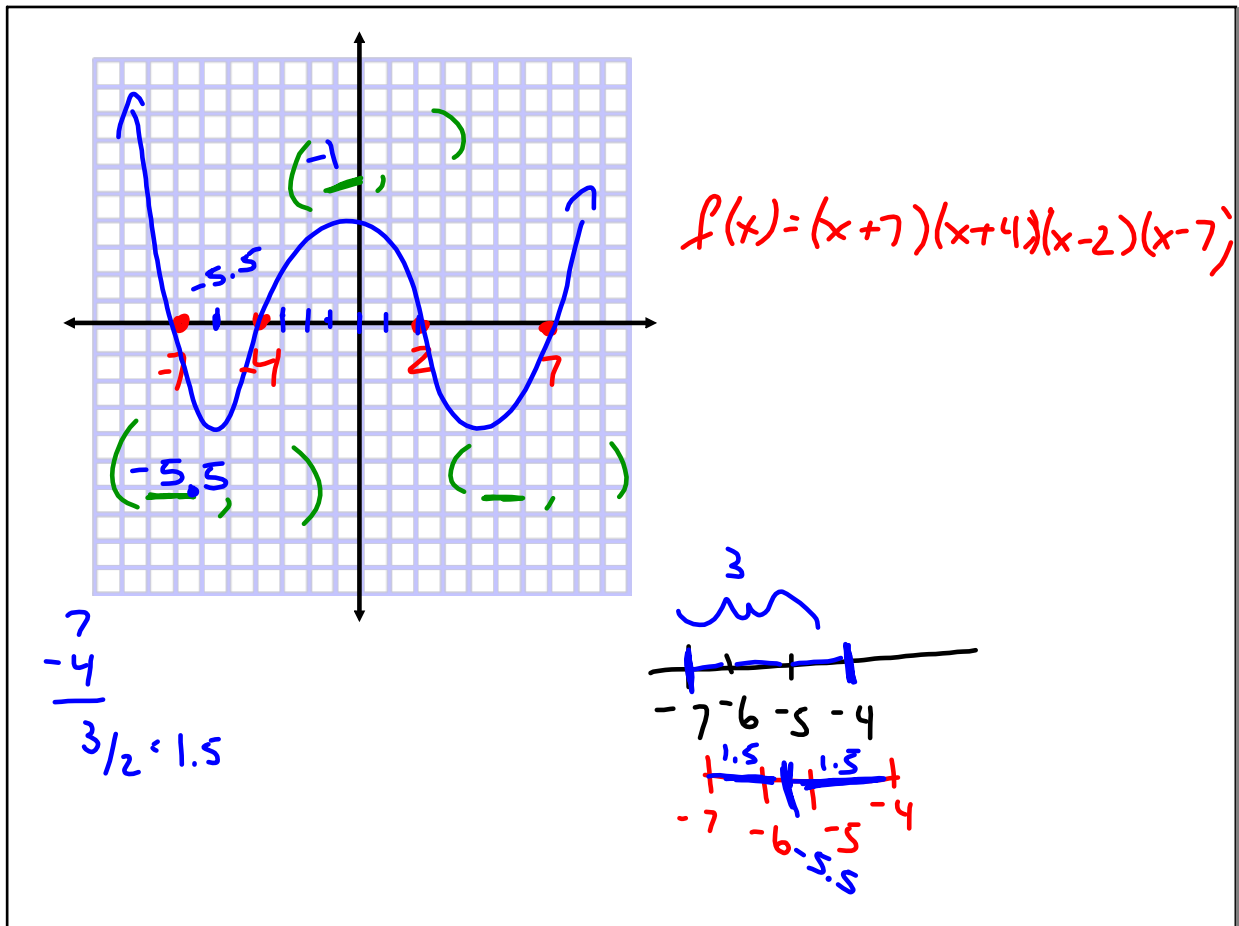
$f(x) = (x+5)(x+2)(x-2)(x-6)$

$f(-3.5) = (-3.5+5)(-3.5+2)(-3.5-2)(-3.5-6)$
 $= (1.5)(-1.5)(-5.5)(-9.5)$
 $= -117.5625$

$f(0) = (0+5)(0+2)(0-2)(0-6)$
 $= (5)(2)(-2)(-6)$
 $= (10)(-2)(-6)$
 $= (20)(-6)$
 $= 120$

$f(4) = -216$
 $(4+5)(4+2)(4-2)(4-6)$

Sep 2-8:12 AM



Sep 4-9:56 AM

① $x^3 - 8x^2 + 11x + 20$ Graph & tell end behavior

② $x^4 + x^3 - 13x^2 - x + 12$

③ $x^4 - 10x^2 + 9$

$x \rightarrow -\infty$ $x \rightarrow +\infty$
 $y \rightarrow$ $y \rightarrow$

Today,
 max: ()
 min: ()

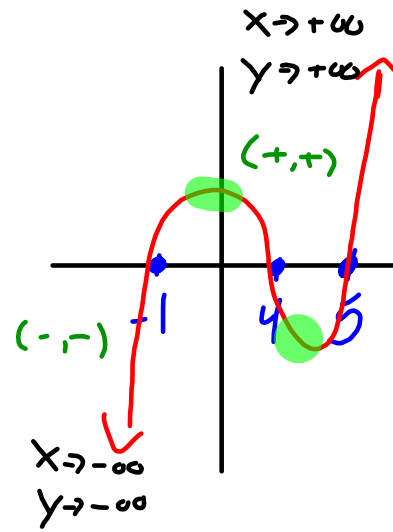
Sep 3-9:00 AM

① $x^3 - 8x^2 + 11x + 20$

$x = 5$ | -8 11 20

↓ 5 -15 -20
 1 -3 -4 0

$x^2 - 3x - 4$
 $(x - 4)(x + 1)$
 $x = 4 \quad x = -1$



Sep 4-12:18 PM

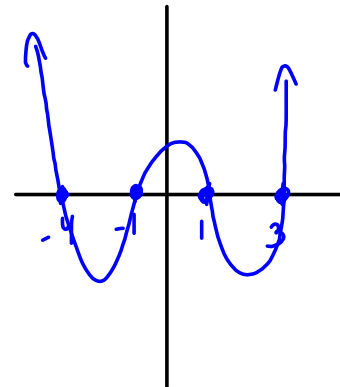
② $x^4 + x^3 - 13x^2 - x + 12$

$p/q = \pm 1, 2, 3, 4, 6, 12$

$x = 1$ | 1 -13 -1 12
 ↓ 1 2 -11 -12
 1 2 -11 -12 0

$x = -1$ | 2 -11 -12
 ↓ -1 -1 +12
 1 1 -12 0

$x^2 + x - 12$
 $(x + 4)(x - 3)$
 $x = -4 \quad x = 3$



Sep 4-12:18 PM

③ $x^4 - 10x^2 + 9$
 $x^4 + 0x^3 - 10x^2 + 0x + 9$
 $P/Q = \pm 1, 3, 9$

$x = 3$

1	0	-10	0	9
↓	3	9	-3	-9
1	3	-1	-3	0

$x = -3$

1	3	-1	-3
↓	-3	0	3
1	0	-1	0

$x^2 - 1 = 0$
 $(x+1)(x-1)$
 $x = -1 \quad x = 1$

$x^2 - 1 = 0$
 $x^2 = 1$
 $x = \pm\sqrt{1}$
 $x = \pm 1$

Sep 4-12:18 PM

To Find max & min

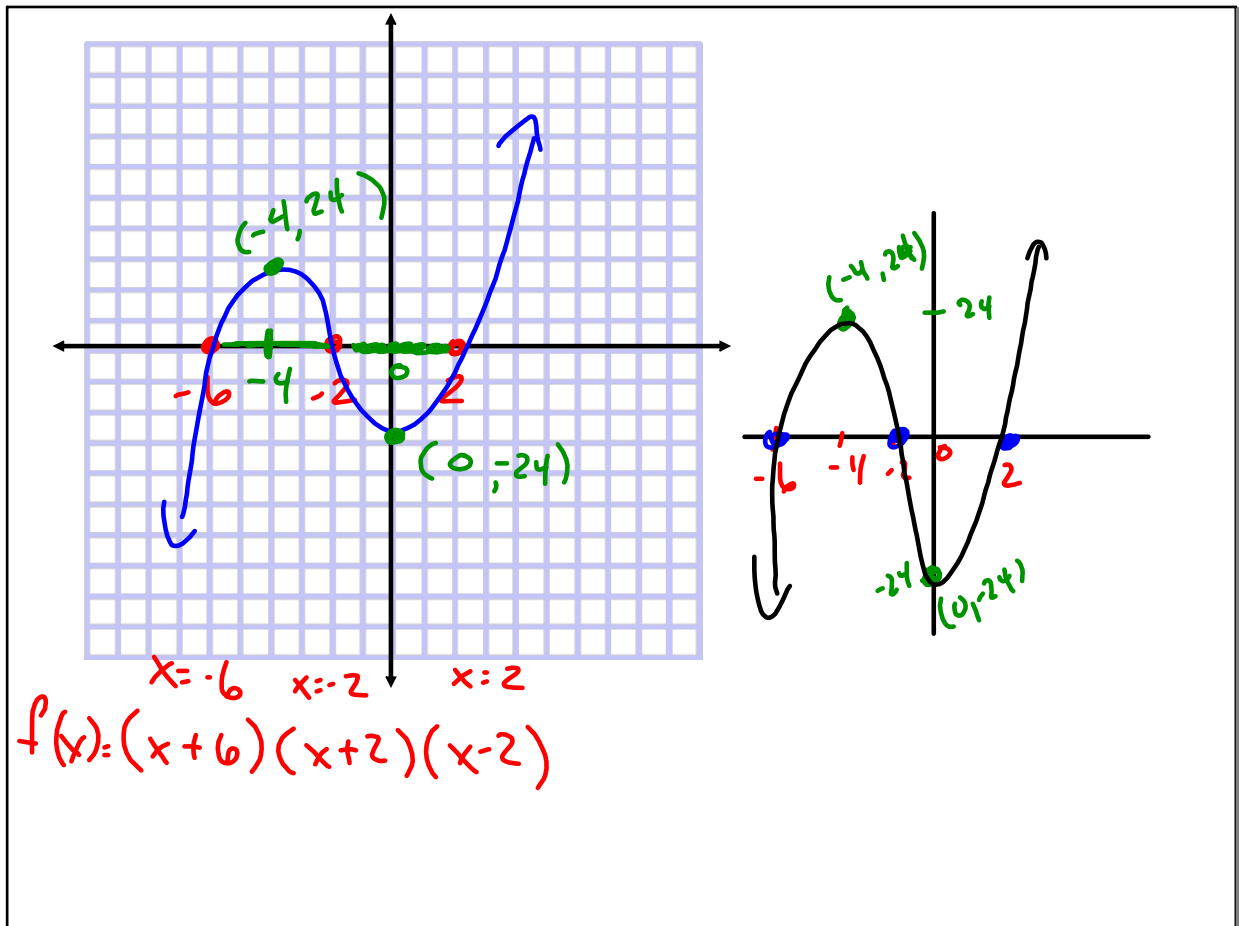
- ① x-intercepts
- ② distance between x-int.
- ③ Find midpoint
- ④ plug into equation

* use the factors from x-int.

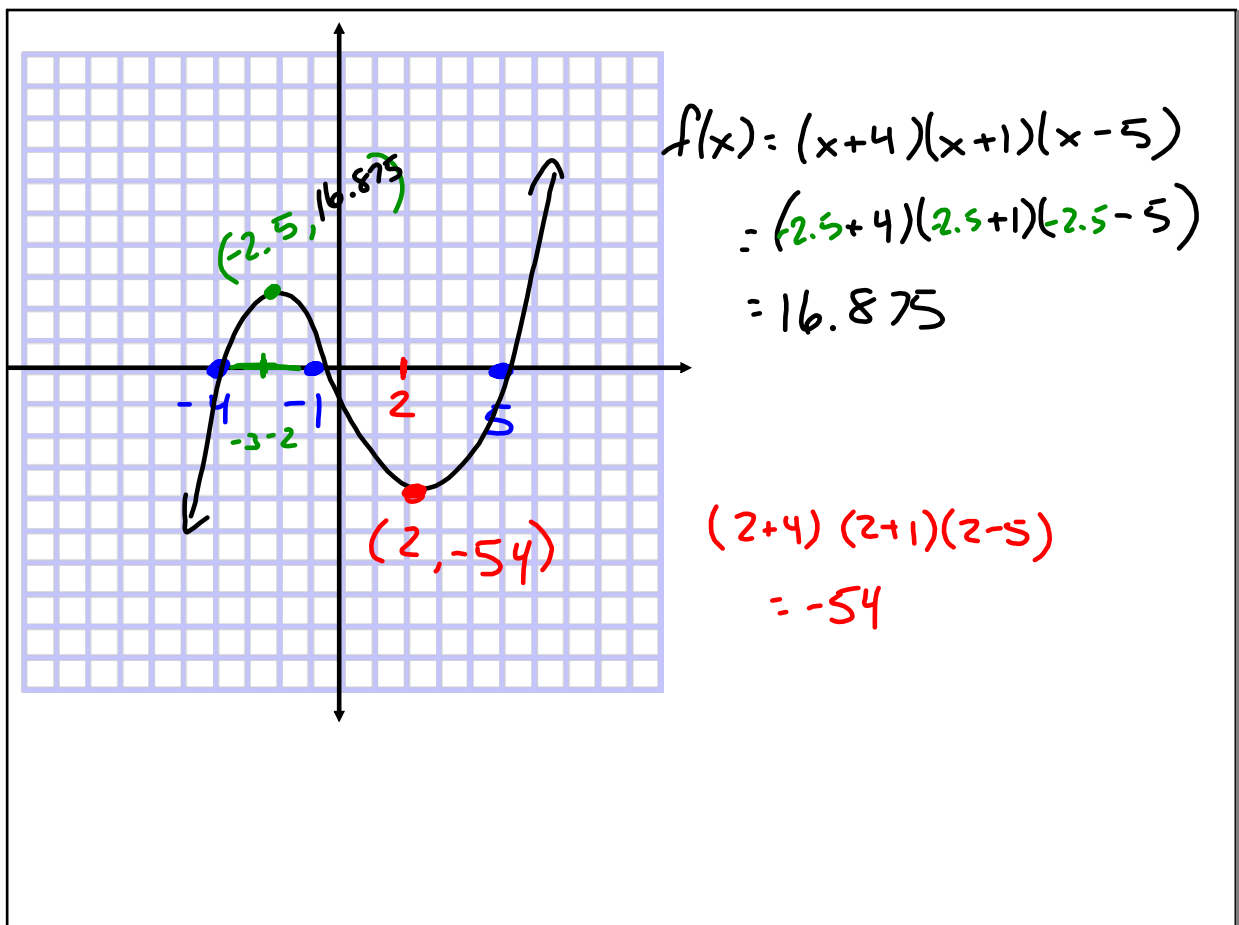
$(-3+6)(-3)(-3-4)$
 $(3)(-3)(-7)$
 $-9(-7)$

$f(x) = -(x+6)(x)(x-4)$
 $f(2) = (2+6)(2)(2-4)$
 $= (8)(2)(-2)$
 $= 16(-2)$
 $= -32$

Sep 4-8:42 AM

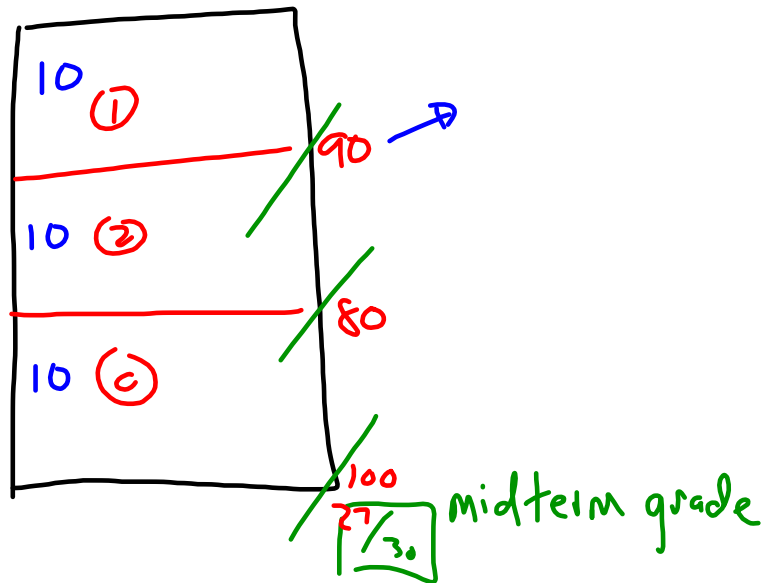


Sep 4-12:42 PM



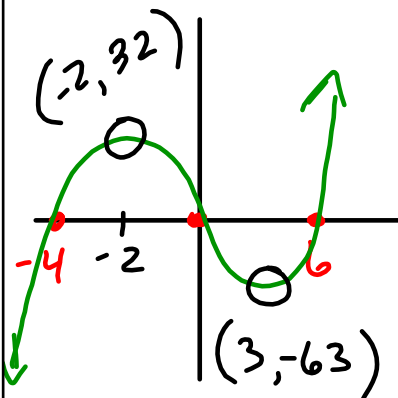
Sep 4-12:49 PM

Re-test \Rightarrow midterm



Sep 8-8:19 AM

Practice



Graph the polynomial
and label the
max. & min.

positive cubic function

$$\textcircled{1} f(x) = (x+4)(x)(x-6)$$

$$\textcircled{2} f(-2) = (-2+4)(-2)(-2-6) = 32$$

$$f(3) = (3+4)(3)(3-6) = -63$$

Sep 8-8:23 AM

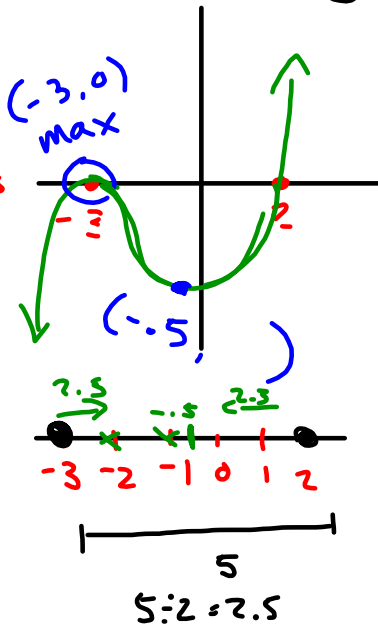
What is multiplicity?

$$5/2 = 2.5$$

words:

how many of the same solutions you have

drawing:



Solutions:

$$f(x) = (x-2)(x+3)^2$$

$$(x-2) = 0$$

$$x = 2$$

$$(x+3)^2 = (x+3)(x+3)$$

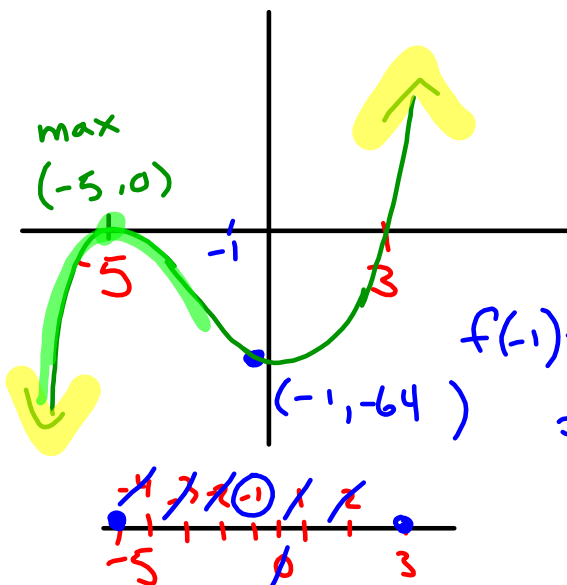
$$x = -3 \quad x = -3$$

multiplicity of 2 of $x = -3$

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graph: $x = 3$

$x = -5$, multiplicity of 2
positive function



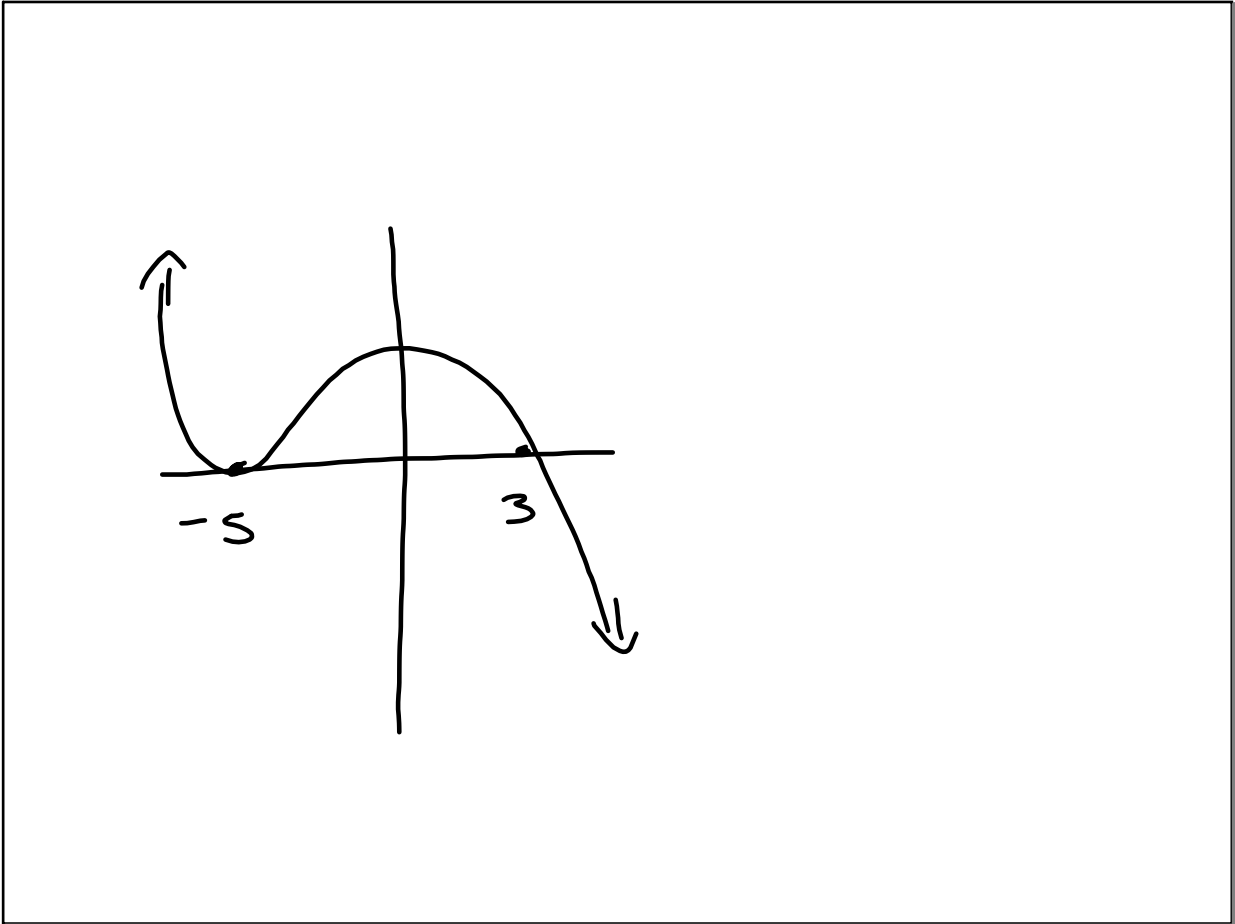
$$f(-1) = (x+5)^2(x-3)$$

$$= (4)^2(-4)$$

$$= 16(-4)$$

$$= -64$$

Sep 8-8:45 AM



Sep 8-8:56 AM

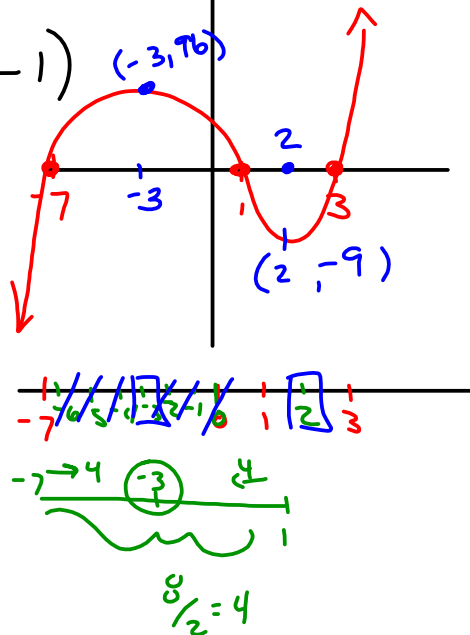
Practice :

graph with
max and min labeled.

$$f(x) = (x-3)(x+7)(x-1)$$

$$\begin{aligned} f(-3) &= (-6)(4)(-4) \\ &= -24(-4) \\ &= 96 \end{aligned}$$

$$\begin{aligned} f(2) &= (-1)(9)(1) \\ &= -9 \end{aligned}$$



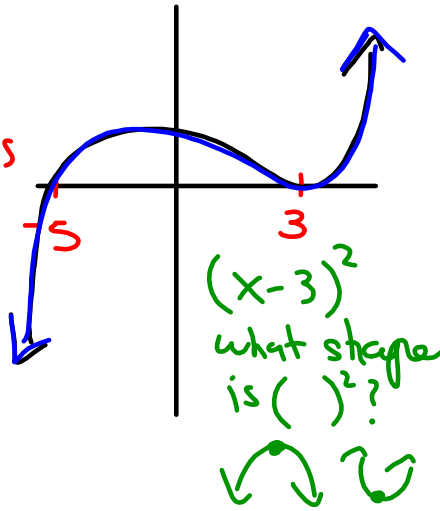
Sep 8-9:11 AM

What is multiplicity?

words

How many of the same solutions do you have.

picture



Equation

$$f(x) = (x-3)^2(x+5)$$

$$(x-3)^2 = (x-3)(x-3)$$

$$x=3 \quad x=3$$

Multiplicity of 2 of $x=3$

$$(x+5)$$

$$x=-5$$

Sep 8-9:26 AM

Graph, xint, max, min.

ALL positive

① $f(x) = (x-2)^2(x+3)(x-4)$

② $x=3$, $x=5$ w/ multiplicity of 2

③ $x=4$, $x=-2$, $x=-2$

④ $f(x) = x(x-4)^2(x+2)^2$

Sep 8-8:57 AM

TEST *solve*

TEST *graph*

TEST *systems*

midterm	
TEST <i>solve</i> -2	80
TEST <i>graph</i> -1	90
TEST <i>systems</i> 0	100
	10

$\frac{27}{30} = 90$ midterm grade

Sep 8-12:55 PM

picture

Equation

$$f(x) = (x-3)^2(x+5)$$

$$(x-3)^2 = (x-3)(x-3)$$

$$x=3 \quad x=3$$

multiplicity of 2 of $x=3$

$$(x+5)$$

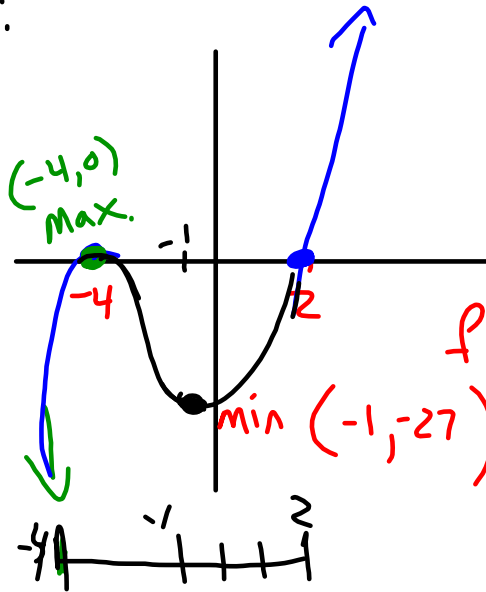
$$x=-5$$

$(x-3)^2$
what is this shape?
or $x=3$

Sep 8-9:32 AM

$$f(x) = (x-2)(x+4)^2$$

Graph :



Find the max & min

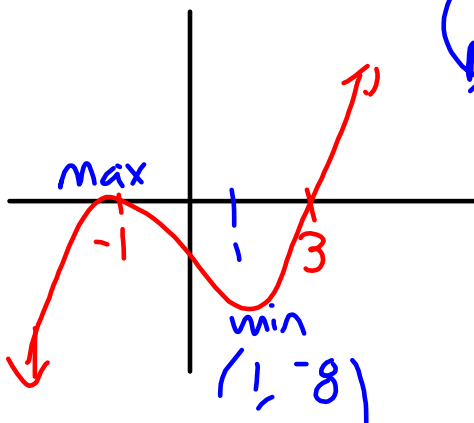
$$\begin{aligned} f(-1) &= (-3)(3)^2 \\ &= -3(9) \\ &= -27 \end{aligned}$$

Sep 8-9:38 AM

Graph

$$\begin{aligned} x &= 3 & x &= -1 \\ x &= -1 & x &= -1 \end{aligned}$$

positive cubic



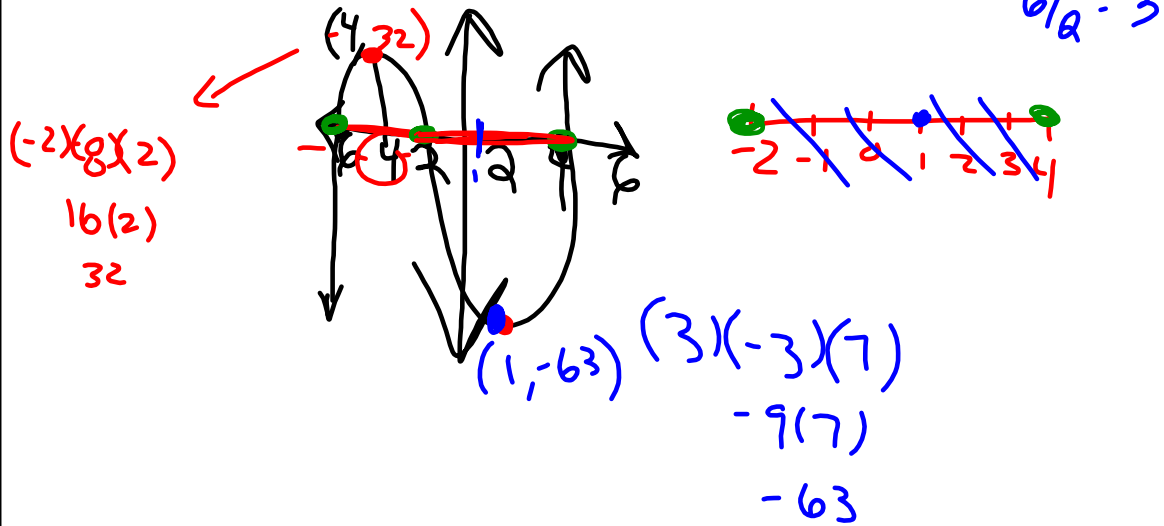
$$\begin{aligned} f(x) &= (x-3)(x+1)^2 \\ f'(x) &= (x+1)^2(x-3) \\ f(1) &= (2)^2(-2) \\ &= 4(-2) \\ &= -8 \end{aligned}$$

Sep 8-9:47 AM

Practice

graph w/ max & min labeled.

$$f(x) = (x+2)(x-4)(x+6)$$



Sep 8-9:39 AM

What is multiplicity:

$$(1+3)^2(1-5)$$

$$(4)^2(-4)$$

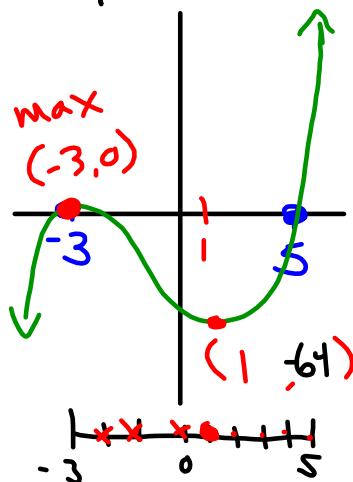
$$16(-4)$$

$$= -64$$

words

tells you that there are more of the same solutions.

picture



Equation/Solutions

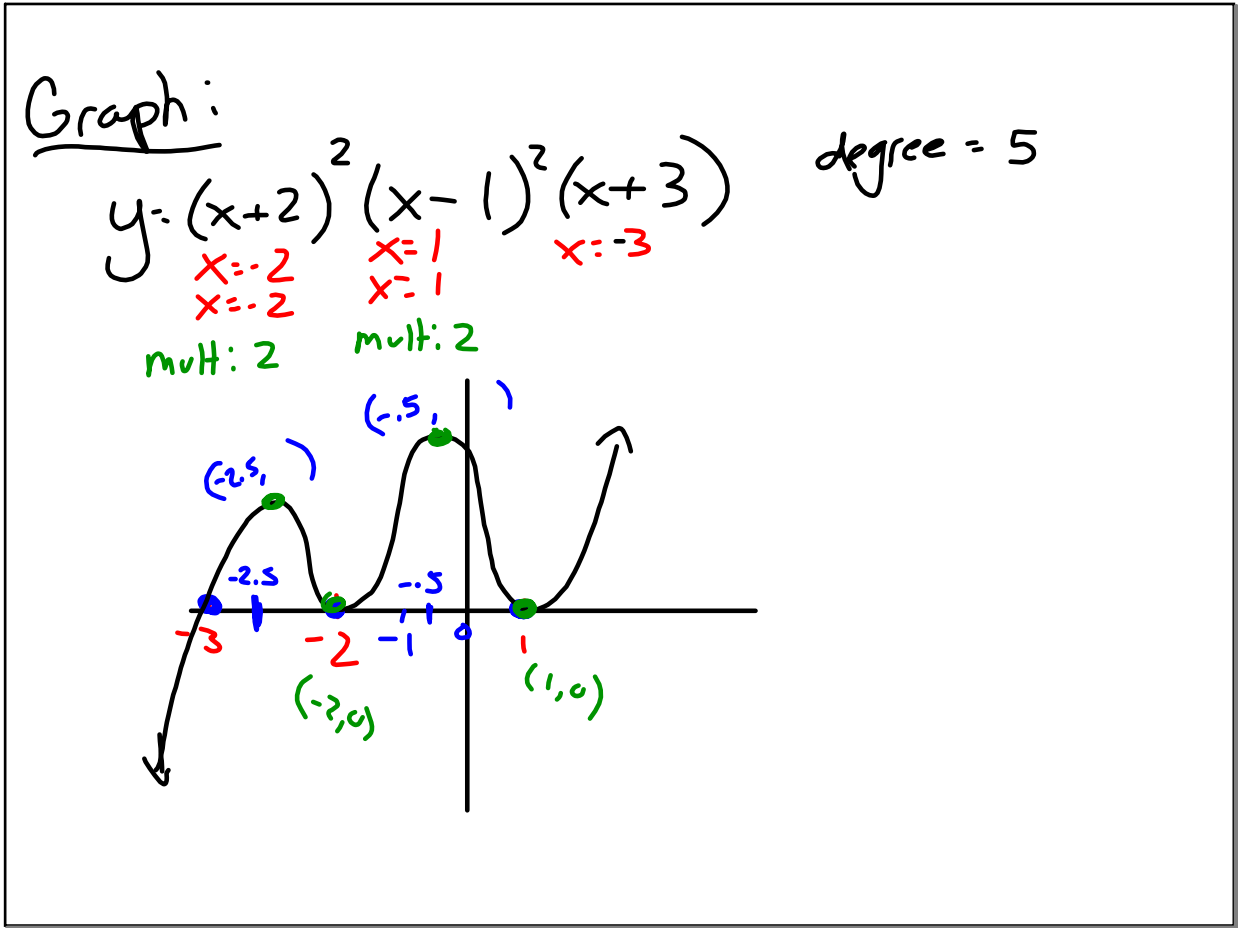
$$f(x) = (x+3)^2(x-5)$$

$$= (x+3)(x+3)(x-5)$$

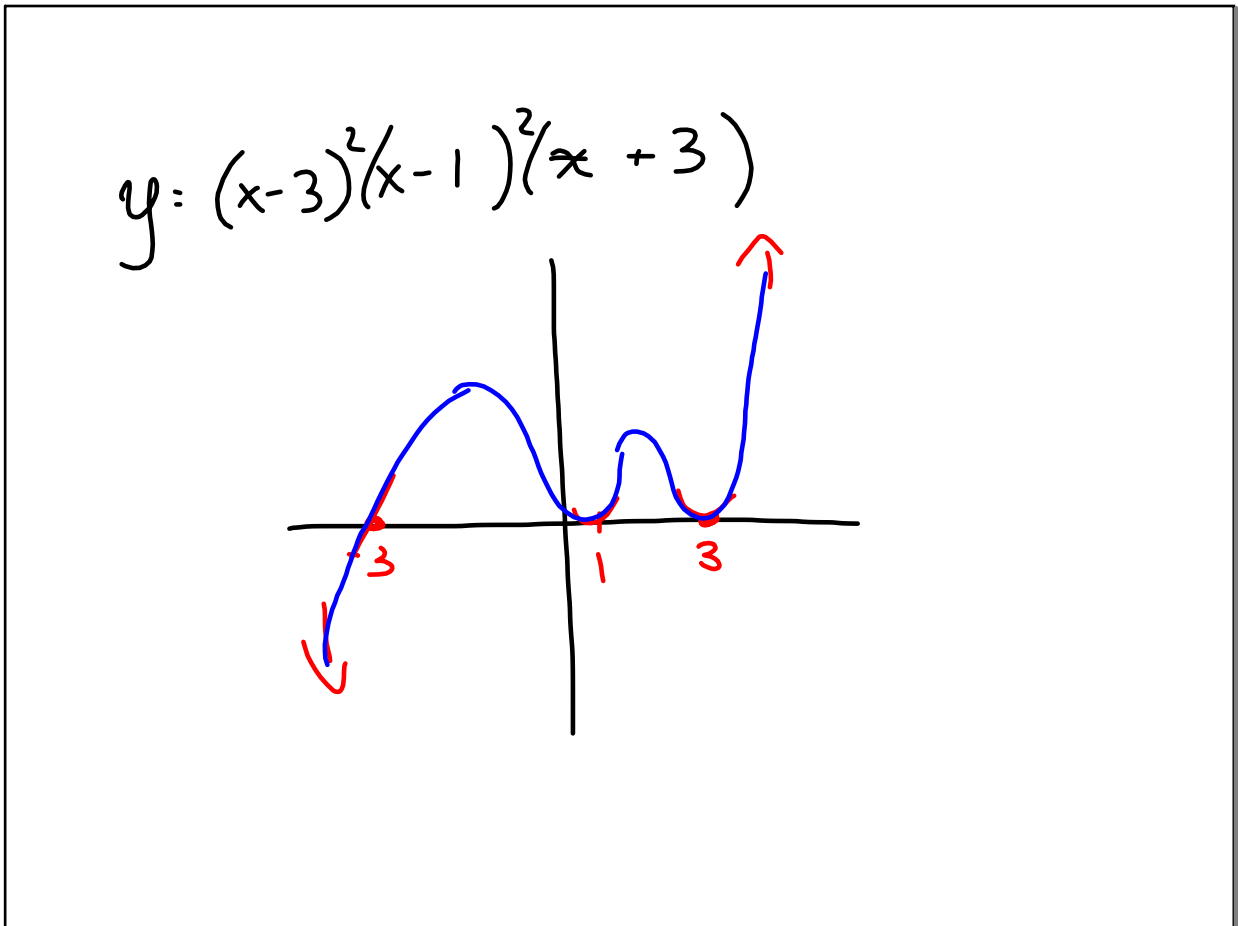
$x = -3$ $x = -3$ $x = 5$

multiplicity of 2 of $x = -3$

Sep 8-12:32 PM



Sep 8-12:42 PM



Sep 8-12:53 PM