

$\frac{3x}{2x+3} = 0$        $\rightarrow$  answer  $x =$

$< 0$        $> 0$        $\leq 0$        $\geq 0$

open circle      closed circle

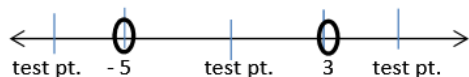
act like  $= 0$   
 $x =$

**Rational Example:**

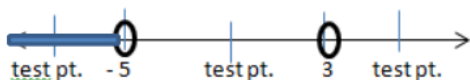
$$\frac{x-3}{x+5} > 0$$

$$x - 3 = 0 \quad x + 5 = 0$$

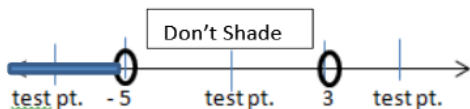
$$x = 3 \quad x = -5$$



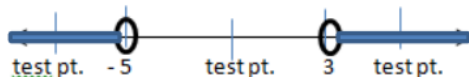
Plug in a number less than -5 such as -6,  $\frac{-6-3}{-6+5} > 0$ ,  $\frac{-9}{-1} = 9$ , is  $9 > 0$  ... YES, so shade this region



Plug in a number between -5 and 3 such as 0...  $\frac{0-3}{0+5} > 0$  ...  $\frac{-3}{5} > 0$  ... is  $\frac{-3}{5} > 0$  ... NO ... Don't shade this region



Plug in a number greater than 3 such as 4 ...  $\frac{4-3}{4+5} > 0$  ...  $\frac{1}{9} > 0$  ... is  $\frac{1}{9} > 0$  ... YES ... Shade this region



**Practice Problems:**

1.  $\frac{x+5}{x-2} > 0$

2.  $\frac{2x-3}{5x} > 0$

3.  $\frac{2x+5}{x-2} + \frac{4x+7}{x-2} > 0$  (hint: First Add the Fractions)

4.  $\frac{x+1}{x-8} < 0$

5.  $\frac{x-8}{x+9} > 0$

6.  $\frac{-2x+7}{x-1} + \frac{4x-3}{x-1} < 0$  (hint: First Add the Fractions)

**Rational Inequalities**

Steps:

1. Move all terms to one side
2. Simplify to 1 Rational Function or Fraction
3. Set top = 0    Set bottom = 0
4. Make a number line
5. Plug in answers from step #3
6. Use test points to get your answers

$$1. \frac{x+5}{x-2} > 0$$

top  $\Rightarrow x+5=0$

bottom  $\Rightarrow x-2=0$

$$x = -5$$

$$x = 2$$

True  
YES  
TP

False  
No  
TP=0

Now...

Where do we shade?

original equation

$$\frac{-6}{-8-2} > 0$$

$$\frac{0}{0-2}$$

$$\frac{3}{3-2}$$

$$\frac{-6+5}{-6-2} > 0$$

$$\frac{5}{-2} > 0$$

$$\frac{3+5}{3-2}$$

$$\frac{-1}{-8} > 0$$

$$\frac{8}{1} > 0$$

$$\frac{1}{8} > 0$$

open circle      open circle

Notes on Notation

○ open circle  
( )

● [ ]

-∞ or +∞  
( )

\* denominator  
Never gets [ ]

Answer:  $(-\infty, -5) \cup (2, \infty)$

$$2. \frac{2x-3}{5x} > 0$$

greater than  $\Rightarrow$  open circle

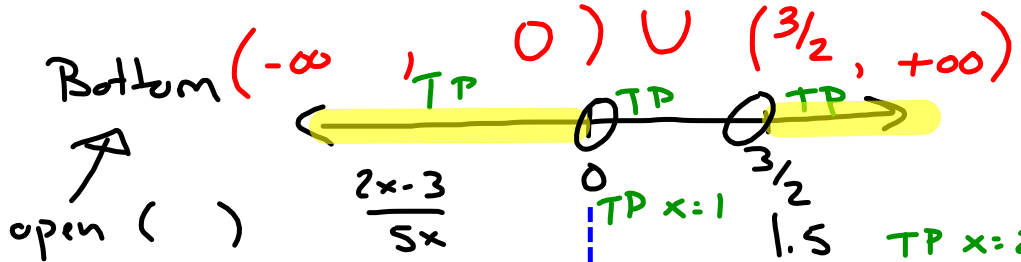
$$2x - 3 = 0$$

$$x = 3/2 \quad \text{Top}$$

answer notation ( )

$$5x = 0$$

$$x = 0$$



open ( )

$$\frac{2x-3}{5x}$$

$$TP = -1$$

$$\frac{2(-1)-3}{5(-1)} > 0$$

$$\frac{-2-3}{-5} > 0$$

$$\frac{-5}{-5} = 1 > 0$$

YES  
True

$$\frac{2x-3}{5x}$$

$$TP \ x=1$$

$$\frac{2(1)-3}{5(1)}$$

$$\frac{2-3}{5}$$

$$\frac{-1}{5} > 0$$

No  
False

$$\frac{2x-3}{5x}$$

$$TP \ x=2$$

$$\frac{2(2)-3}{5(2)}$$

$$\frac{4-3}{10}$$

$$\frac{1}{10} > 0$$

YES  
True

3.  $\frac{2x+5}{x-2} + \frac{4x+7}{x-2} > 0$  (hint: First Add the Fractions)

$$\frac{2x+5+4x+7}{x-2} > 0$$

$$\frac{6x+12}{x-2} > 0$$

open circle

Top  $\Rightarrow 6x+12=0$   
 $6x = -12$   
 $x = -2$

Bottom  $\Rightarrow x-2=0$   
 $x = 2$

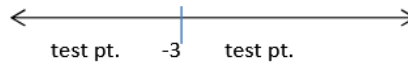
$(-\infty, -2)$   $(2, \infty)$

Interval	Test Point (TP)	Substitution	Result
$(-\infty, -2)$	$x = -3$	$\frac{6(-3)+12}{-3-2} > 0$ $\frac{-18+12}{-5} > 0$ $\frac{-6}{-5} > 0$ $\frac{6}{5} > 0$	YES
$(-2, 2)$	$x = 0$	$\frac{6(0)+12}{0-2} > 0$ $\frac{12}{-2} > 0$ $-6 > 0$	No
$(2, \infty)$	$x = 3$	$\frac{6(3)+12}{3-2} > 0$ $\frac{18+12}{1} > 0$ $\frac{30}{1} > 0$ $30 > 0$	YES

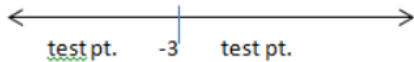
Radical Inequalities

Example:  $\sqrt{x+3} > 0$  ... solve for  $x$ . We change the inequality sign to an equals to solve  $\sqrt{x+3} = 0$

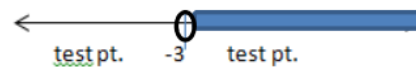
$(\sqrt{x+3})^2 = 0^2$  ...  $x+3 = 0$  ... so  $x = -3$



test points  $x = -4$  and try  $x = 0$  to see where to shade



$\sqrt{-4+3} > 0$  ...  $\sqrt{-1} > 0$  ... not true, so don't shade this region.



$\sqrt{0+3} > 0$  ...  $\sqrt{3} > 0$  ... this is true, so shade this region.

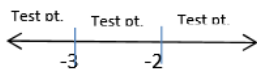
Practice:

1.  $\sqrt{3x+2} > 0$                       2.  $\sqrt{4x-7} > 0$                       3.  $\sqrt{x+8} > 0$

4.  $\sqrt{4x+8} < 0$                       5.  $\sqrt{\frac{27x}{3}} > 0$                       6.  $\sqrt{\frac{6}{18x}} < 0$

Now take this same process to make the number lines for polynomials

1.  $(x+3)(x+2) > 0$                       2.  $(x-4)(x+7) < 0$                       3.  $x^2 + 7x + 12 > 0$



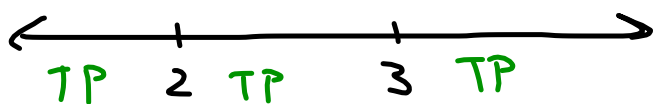
4.  $x^2 - 5x + 6 < 0$                       6.  $x^2 + 4x - 12 < 0$                       7.  $2x^2 - 7x - 3 < 0$

$$4. x^2 - 5x + 6 < 0$$

must Factor

$$(x - 3)(x - 2) < 0$$

$$x = 3 \quad x = 2$$



$$2. (x - 4)(x + 7) < 0$$

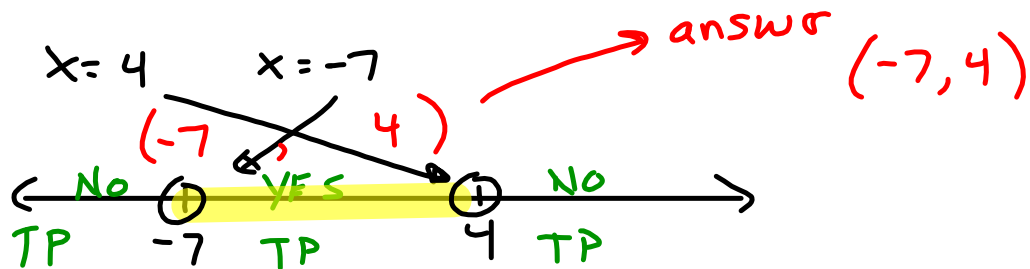
Step 1)

$$(x - 4)(x + 7) = 0$$

$$x - 4 = 0 \quad x + 7 = 0$$

$$x = 4$$

$$x = -7$$



$$x = -8$$

$$(-8 - 4)(-8 + 7) < 0$$

$$(-12)(-1) < 0$$

$$12 < 0$$

No ... False

$$x = 0$$

$$(0 - 4)(0 + 7) < 0$$

$$-4(7) < 0$$

$$-28 < 0$$

YES

$$x = 5$$

$$(5 - 4)(5 + 7) < 0$$

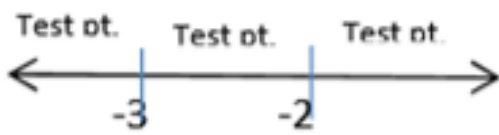
$$(1)(12) < 0$$

$$12 < 0$$

No ... False



1.  $(x + 3)(x + 2) > 0$



4.  $x^2 - 5x + 6 < 0$

1.  $\sqrt{3x+2} > 0$

Step 1)

Set =

$$\sqrt{3x+2} = 0$$

Solve for x

$$(\sqrt{3x+2})^2 = (0)^2$$

$$3x+2 = 0$$

$$3x = -2$$

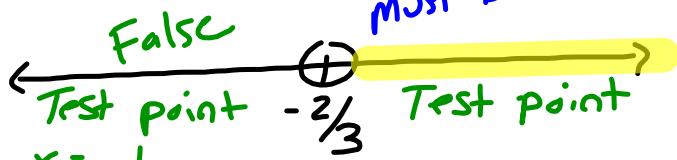
$$x = -\frac{2}{3}$$

$$x = -.67$$

Step 2)

$(-\frac{2}{3}, \infty)$

Must Be true.

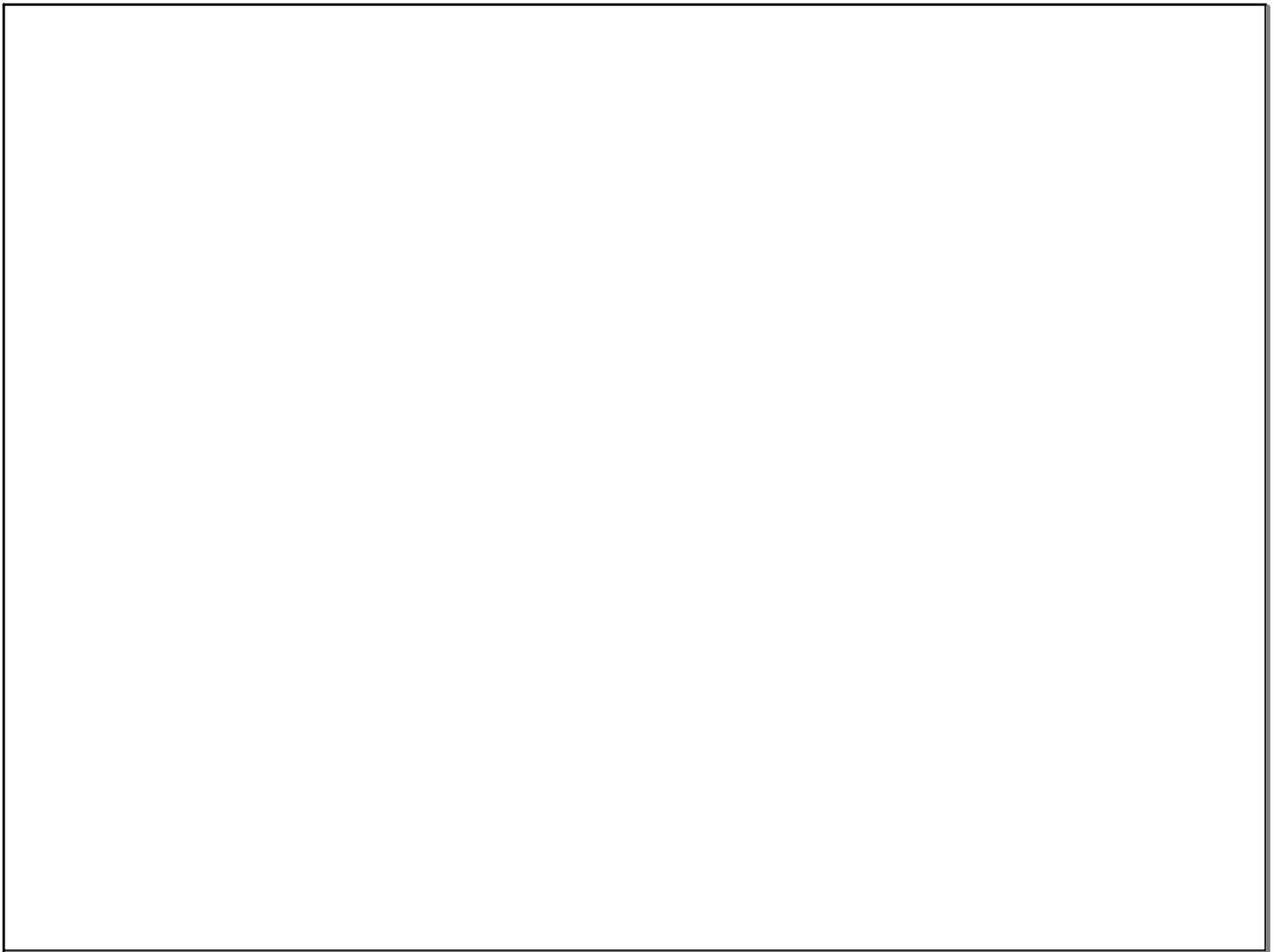


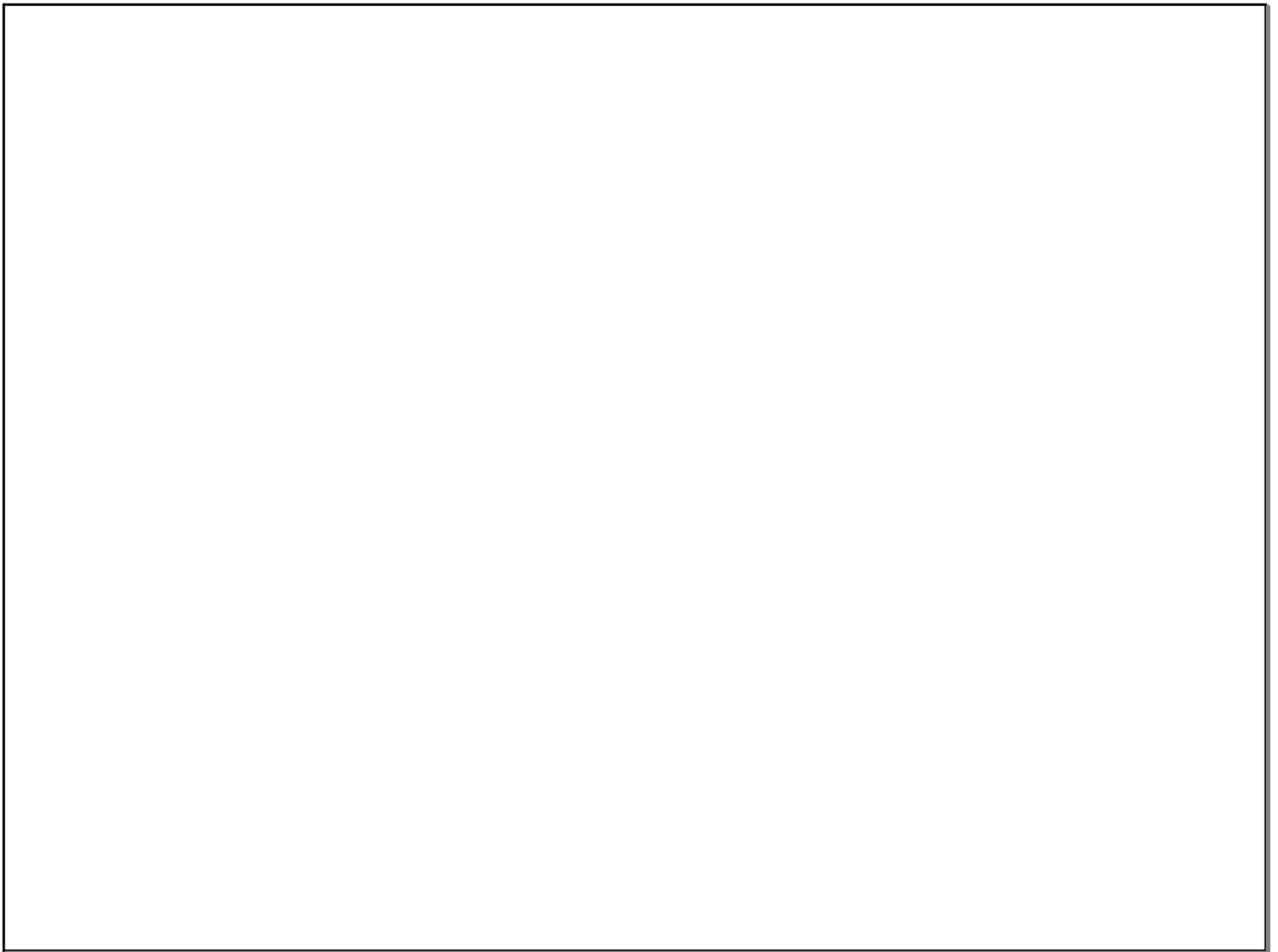
$$\sqrt{3(-1)+2} > 0$$

$$\sqrt{-3+2} > 0$$

$$\sqrt{-1} > 0$$

imaginary #  
No!!



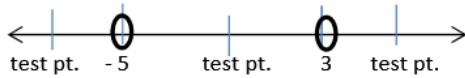


**Rational Example:**

$$\frac{x-3}{x+5} > 0$$

$$x-3=0 \quad x+5=0$$

$$x=3 \quad x=-5$$



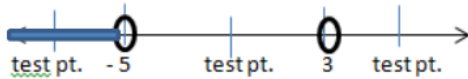
*Rational Function* →

**Rational Inequalities**

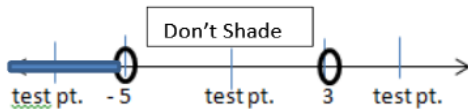
Steps:

1. Move all terms to one side
2. Simplify to 1 Rational Function or Fraction
3. Set top = 0    Set bottom = 0
4. Make a number line
5. Plug in answers from step #3
6. Use test points to get your answers

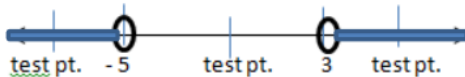
Plug in a number less than -5 such as -6,  $\frac{-6-3}{-6+5} > 0$ ,  $\frac{-9}{-1} = 9$ , is  $9 > 0$  ... YES, so shade this region



Plug in a number between -5 and 3 such as 0...  $\frac{0-3}{0+5} > 0$  ...  $\frac{-3}{5} > 0$  ... is  $\frac{-3}{5} > 0$  ... NO ... Don't shade this region



Plug in a number greater than 3 such as 4 ...  $\frac{4-3}{4+5} > 0$  ...  $\frac{1}{9} > 0$  ... is  $\frac{1}{9} > 0$  ... YES ... Shade this region



**Practice Problems:**

1.  $\frac{x+5}{x-2} > 0$       2.  $\frac{2x-3}{5x} > 0$       3.  $\frac{2x+5}{x-2} + \frac{4x+7}{x-2} > 0$  (hint: First Add the Fractions)

4.  $\frac{x+1}{x-8} < 0$       5.  $\frac{x-8}{x+9} > 0$       6.  $\frac{-2x+7}{x-1} + \frac{4x-3}{x-1} < 0$  (hint: First Add the Fractions)

Inequalities

Equals

$<$  } open circle

$>$  }  parenthesis ( )

$\leq$  } closed circle

$\geq$  }  bracket [ ]

=  $x = \#$

$3x + 2 = 0$   
 $x = -2/3$

$$1. \frac{x+5}{x-2} > 0$$

← Inequality ">" open circle

Top  $x + 5 = 0$

Bottom  $x - 2 = 0$

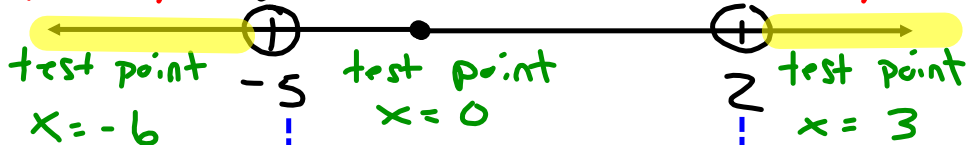
$$x = -5$$

$$x = 2$$

↻ always open circle

Answer  $(-\infty, -5)$

$(2, \infty)$



Original equation

$$\frac{x+5}{x-2} > 0$$

$$\frac{x+5}{x-2} > 0$$

$$\frac{x+5}{x-2} > 0$$

$$\frac{-6+5}{-6-2} > 0$$

$$\frac{0+5}{0-2} > 0$$

$$\frac{3+5}{3-2} > 0$$

$$\frac{-1}{-8} > 0$$

$$\frac{5}{-2} > 0$$

$$\frac{8}{1} > 0$$

$$\frac{1}{8} > 0$$

No  
False

$$8 > 0 \text{ YES}$$

YES

True

Answer:  $(-\infty, -5) \cup (2, \infty)$

$$2. \frac{2x-3}{5x} > 0 \quad \leftarrow \text{open circle}$$

$$\text{Top: } 2x-3 = 0$$

$$2x = 3$$

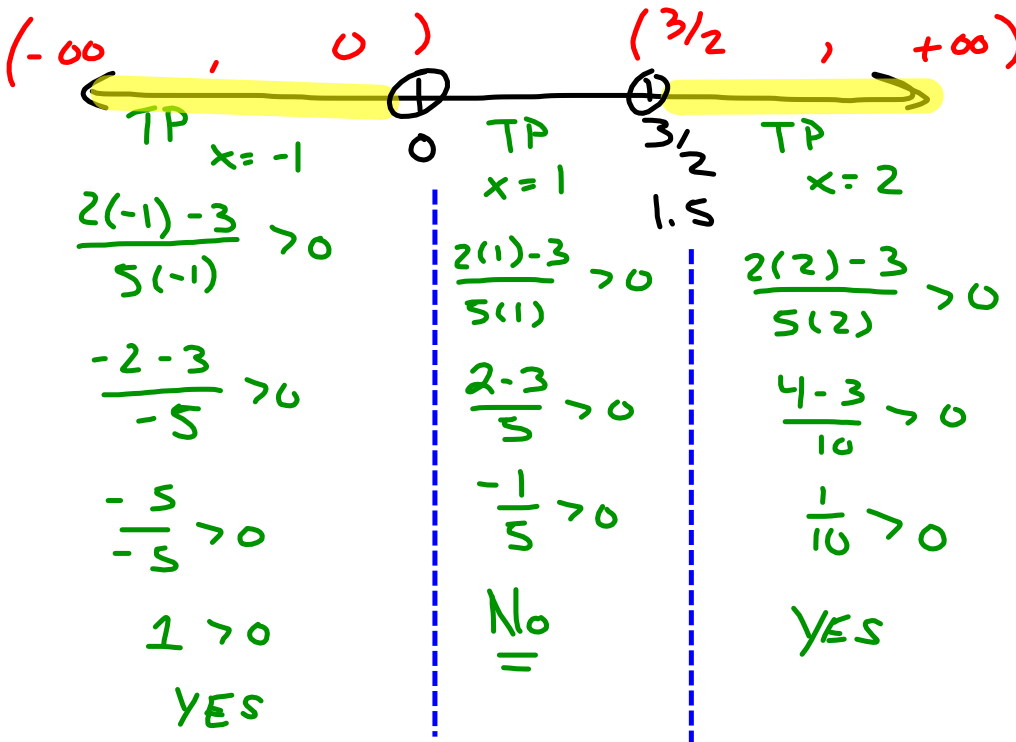
$$x = 3/2$$

$$x = 1.5$$

$$\text{Bottom: } 5x = 0$$

$$x = 0$$

↑  
open  
circle





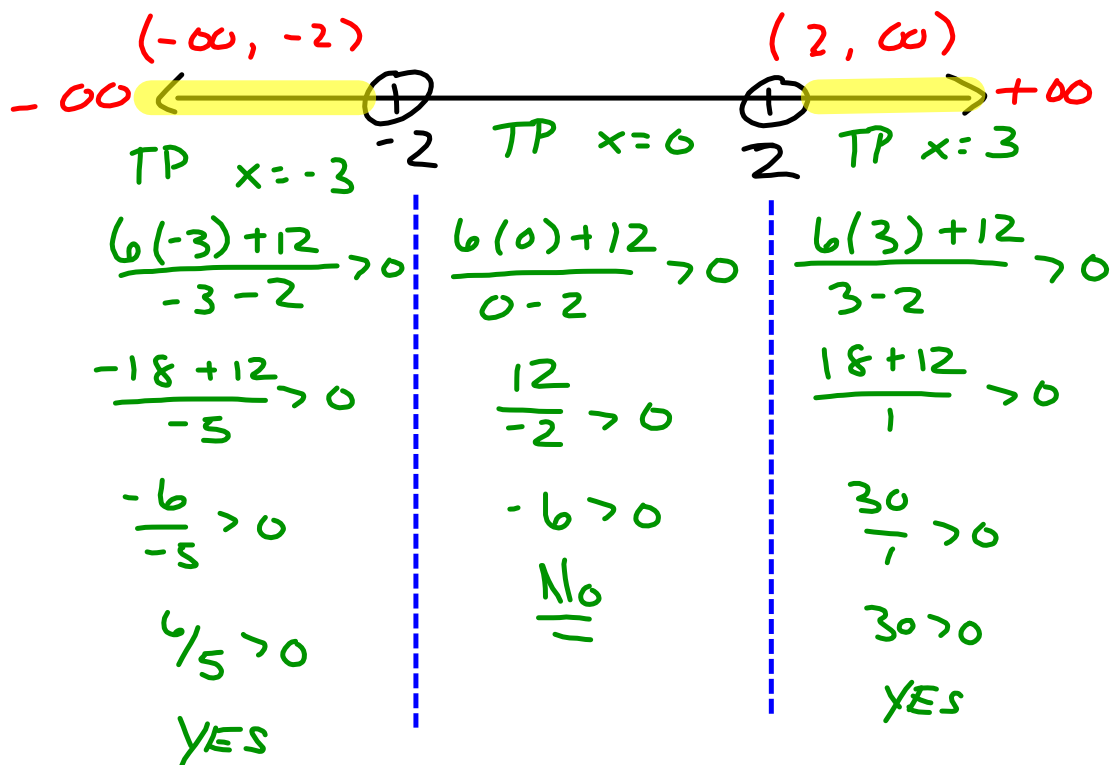
$$3. \frac{2x+5}{x-2} + \frac{4x+7}{x-2} > 0 \quad (\text{hint: First Add the Fractions})$$

$$\frac{2x+5+4x+7}{x-2} > 0$$

$$\frac{6x+12}{x-2} > 0$$

$$\begin{aligned} \text{Top: } 6x+12 &= 0 \\ 6x &= -12 \\ x &= -2 \end{aligned}$$

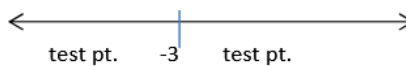
$$\begin{aligned} \text{Bottom: } x-2 &= 0 \\ x &= 2 \end{aligned}$$



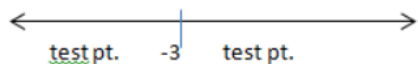
Radical Inequalities

Example:  $\sqrt{x+3} > 0$  ... solve for  $x$ . We change the inequality sign to an equals to solve  $\sqrt{x+3} = 0$

$(\sqrt{x+3})^2 = 0^2$  ...  $x+3 = 0$  ... so  $x = -3$



test points  $x = -4$  and try  $x = 0$  to see where to shade



$\sqrt{-4+3} > 0$  ...  $\sqrt{-1} > 0$  ... not true, so don't shade this region.



$\sqrt{0+3} > 0$  ...  $\sqrt{3} > 0$  ... this is true, so shade this region.

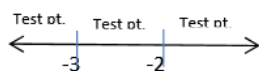
Practice:

- 1.  $\sqrt{3x+2} > 0$
- 2.  $\sqrt{4x-7} > 0$
- 3.  $\sqrt{x+8} > 0$

- 4.  $\sqrt{4x+8} < 0$
- 5.  $\sqrt{\frac{27x}{3}} > 0$
- 6.  $\sqrt{\frac{6}{18x}} < 0$

Now take this same process to make the number lines for polynomials

- 1.  $(x+3)(x+2) > 0$
- 2.  $(x-4)(x+7) < 0$
- 3.  $x^2 + 7x + 12 > 0$



- 4.  $x^2 - 5x + 6 < 0$
- 6.  $x^2 + 4x - 12 < 0$
- 7.  $2x^2 - 7x - 3 < 0$

2.  $(x - 4)(x + 7) < 0$

Steps:

$(x - 4)(x + 7) = 0$

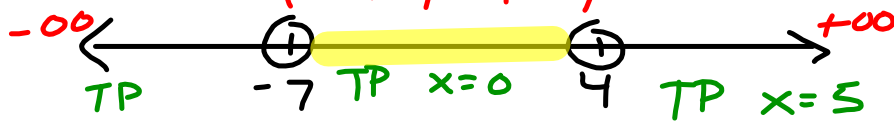
$x - 4 = 0$      $x + 7 = 0$

$x = 4$      $x = -7$

↑  
" < " open circles

answer

$(-7, 4)$



$x = -8$

$(-8 - 4)(-8 + 7) < 0$

$(-12)(-1) < 0$

$12 < 0$

No

$(0 - 4)(0 + 7) < 0$

$-4(7) < 0$

$-28 < 0$

YES

$(5 - 4)(5 + 7) < 0$

$(1)(12) < 0$

$12 < 0$

No

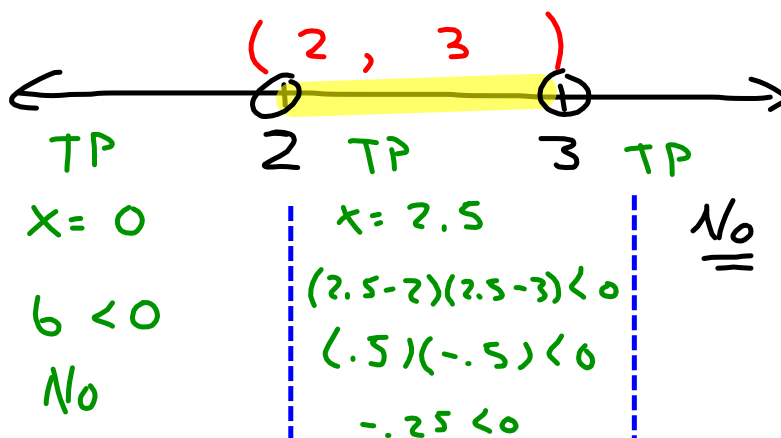
$$\begin{array}{l} 1 \cdot 6 \quad -1 \cdot -6 \\ 2 \cdot 3 \quad -2 \cdot -3 \end{array}$$

4.  $x^2 - 5x + 6 < 0$

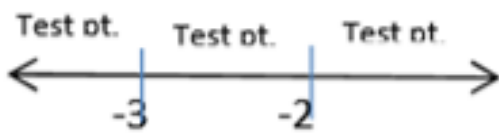
Factor  $(x - 2)(x - 3) < 0$

$$x - 2 = 0 \quad x - 3 = 0$$

$$x = 2 \quad x = 3$$



1.  $(x + 3)(x + 2) > 0$



4.  $x^2 - 5x + 6 < 0$

1.  $\sqrt{3x+2} > 0$

↑  
change to =

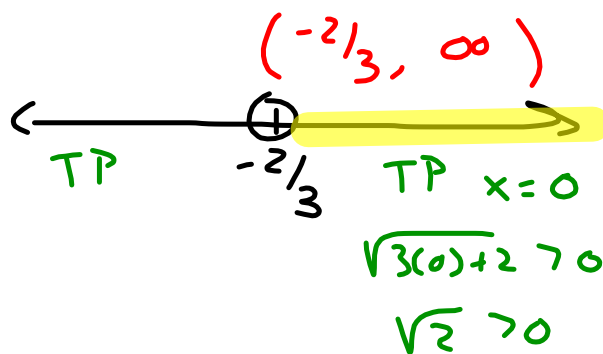
Solve for x

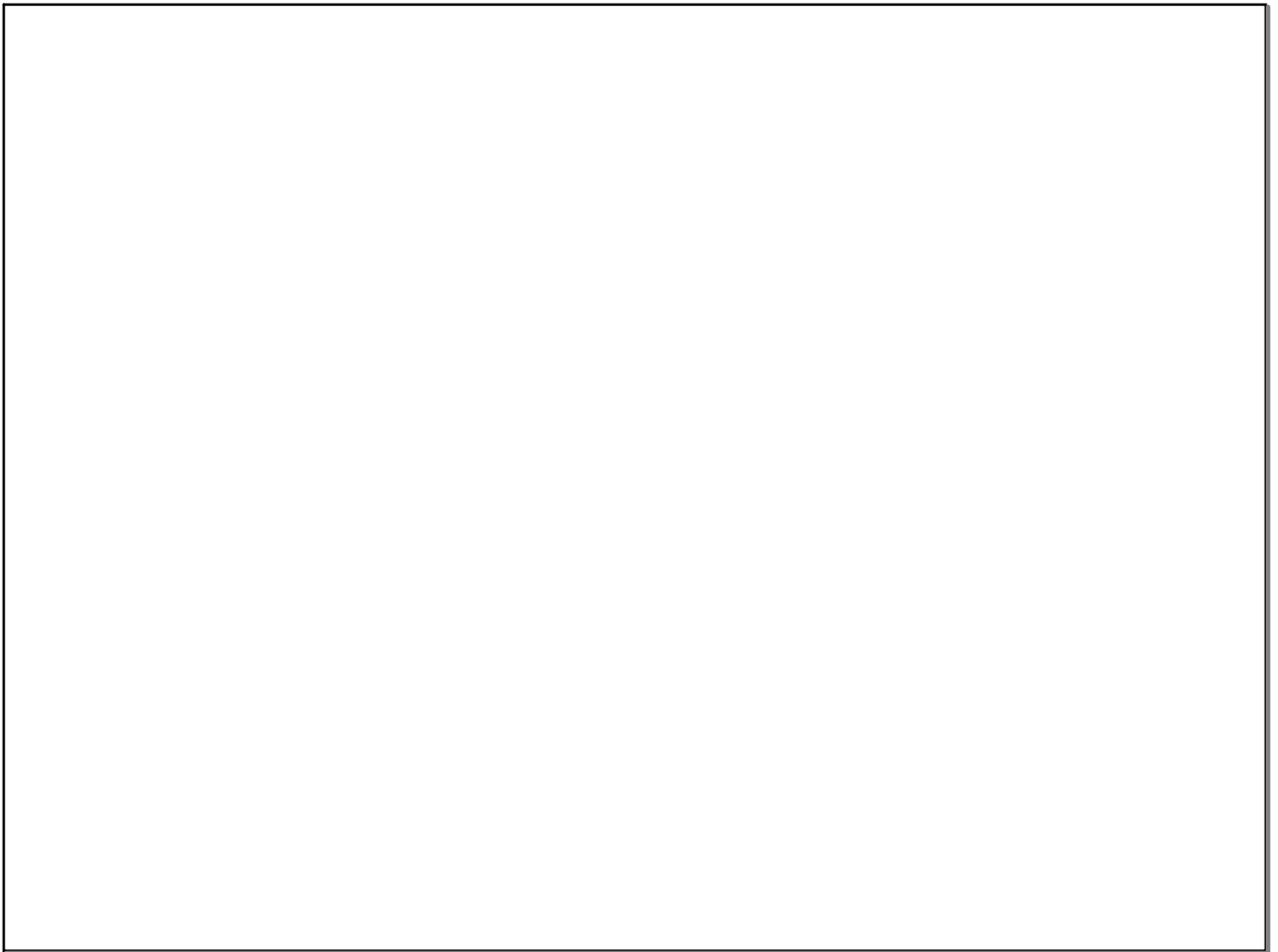
$$(\sqrt{3x+2})^2 = (0)^2$$

$$3x+2 = 0$$

$$3x = -2$$

$$x = -\frac{2}{3}$$





## Inequalities

$=$

$<$

$>$

$\leq$

$\geq$

} open circle & parenthesis  
number line  
written answer

} closed circles & brackets  
number line  
written answers

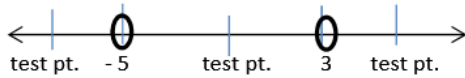


**Rational Example:**

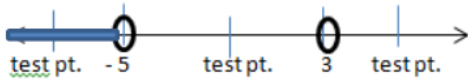
$$\frac{x-3}{x+5} > 0$$

$$x-3=0 \quad x+5=0$$

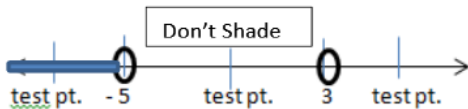
$$x=3 \quad x=-5$$



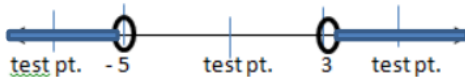
Plug in a number less than -5 such as -6,  $\frac{-6-3}{-6+5} > 0$ ,  $\frac{-9}{-1} = 9$ , is  $9 > 0$  ... YES, so shade this region



Plug in a number between -5 and 3 such as 0...  $\frac{0-3}{0+5} > 0$  ...  $\frac{-3}{5} > 0$  ... is  $\frac{-3}{5} > 0$  ... NO ... Don't shade this region



Plug in a number greater than 3 such as 4 ...  $\frac{4-3}{4+5} > 0$  ...  $\frac{1}{9} > 0$  ... is  $\frac{1}{9} > 0$  ... YES ... Shade this region



**Practice Problems:**

1.  $\frac{x+5}{x-2} > 0$       2.  $\frac{2x-3}{5x} > 0$       3.  $\frac{2x+5}{x-2} + \frac{4x+7}{x-2} > 0$  (hint: First Add the Fractions)

4.  $\frac{x+1}{x-8} < 0$       5.  $\frac{x-8}{x+9} > 0$       6.  $\frac{-2x+7}{x-1} + \frac{4x-3}{x-1} < 0$  (hint: First Add the Fractions)

**Rational Inequalities**

Steps:

1. Move all terms to one side
2. Simplify to 1 Rational Function or Fraction
3. Set top = 0    Set bottom = 0
4. Make a number line
5. Plug in answers from step #3
6. Use test points to get your answers

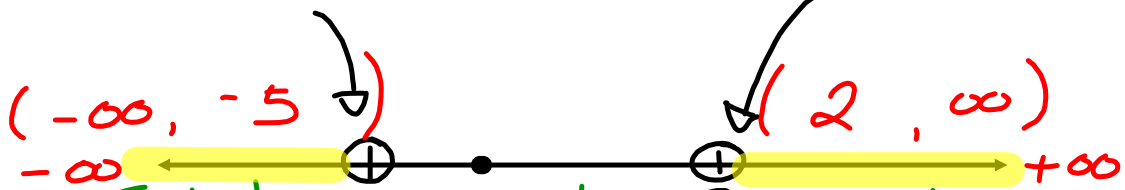
*Rational Function*

1.  $\frac{x+5}{x-2} > 0$  " $>$ " " $>$ " top will be open

Always open circle

top:  $x+5=0$   
 $x = -5$

bottom:  $x-2=0$   
 $x = 2$



Test pt.  
 $x = -6$

$$\frac{-6+5}{-6-2} > 0$$

$$\frac{-1}{-8} > 0$$

$$\frac{1}{8} > 0$$

True  
 YES shade

Test pt.  
 $x = 0$

$$\frac{0+5}{0-2} > 0$$

$$\frac{5}{-2} > 0$$

False  
 Don't shade

test pt.  
 $x = 3$

$$\frac{3+5}{3-2} > 0$$

$$\frac{8}{1} > 0$$

True  
 Yes shade

Answer:  $(-\infty, -5) \cup (2, \infty)$

$$2. \frac{2x-3}{5x} > 0$$

Top:  $2x - 3 = 0$

$$2x = 3$$

$$x = \frac{3}{2}$$

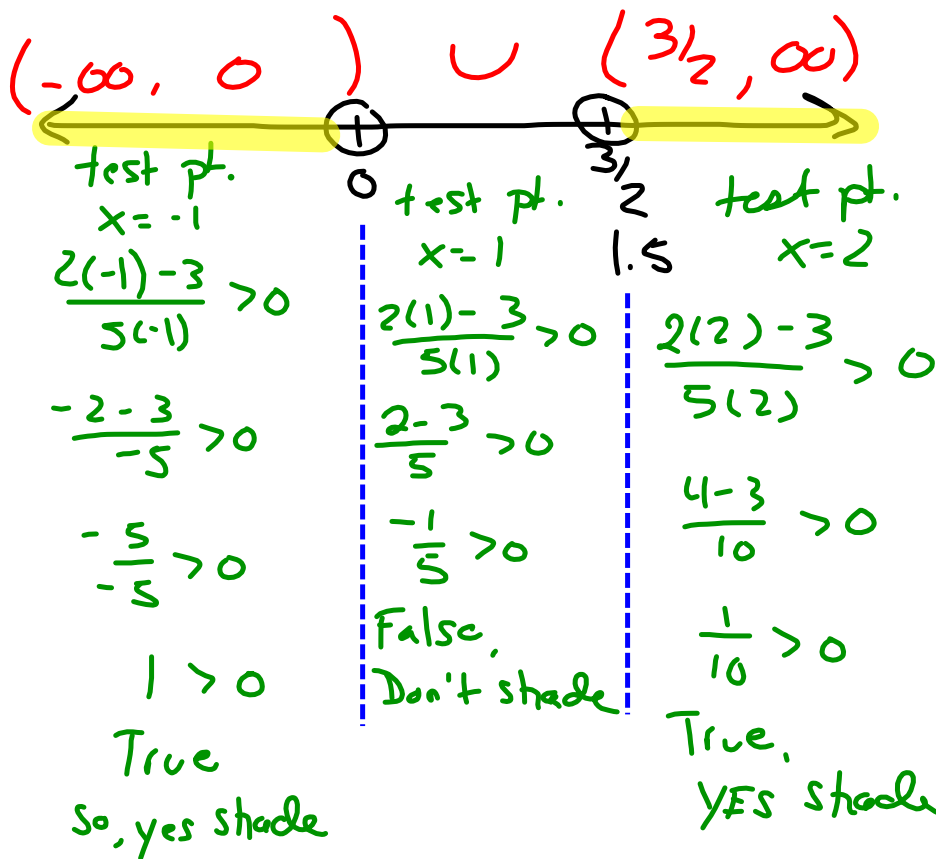
$$x = 1.5$$

Bottom:  $5x = 0$

$$x = 0$$

Always open circle

">"  
open circle



3.  $\frac{2x+5}{x-2} + \frac{4x+7}{x-2} > 0$  (hint: First Add the Fractions)

$$\frac{2x+5+4x+7}{x-2} > 0$$

$$\frac{6x+12}{x-2} > 0$$

Top:  $6x+12=0$

$$6x = -12$$

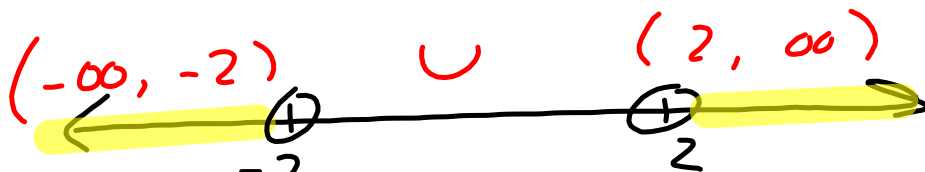
$$x = -2$$

">"  
open  
circle

Bottom:  $x-2=0$

$$x = 2$$

Always  
open  
circle



test pt.

$$x = -3$$

$$\frac{6(-3)+12}{-3-2} > 0$$

$$\frac{-18+12}{-5} > 0$$

$$\frac{-6}{-5} > 0$$

$$\frac{6}{5} > 0$$

True,

Yes shade

test pt.

$$x = 0$$

$$\frac{6(0)+12}{0-2} > 0$$

$$\frac{12}{-2} > 0$$

$$-6 > 0$$

False

don't shade

test pt.

$$x = 3$$

$$\frac{6(3)+12}{3-2} > 0$$

$$\frac{18+12}{1} > 0$$

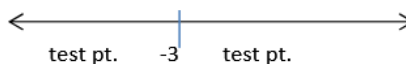
$$30 > 0$$

True, yes shade

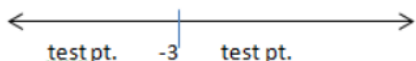
Radical Inequalities

Example:  $\sqrt{x+3} > 0$  ... solve for  $x$ . We change the inequality sign to an equals to solve  $\sqrt{x+3} = 0$

$(\sqrt{x+3})^2 = 0^2$  ...  $x+3 = 0$  ... so  $x = -3$



test points  $x = -4$  and try  $x = 0$  to see where to shade



$\sqrt{-4+3} > 0$  ...  $\sqrt{-1} > 0$  ... not true, so don't shade this region.



$\sqrt{0+3} > 0$  ...  $\sqrt{3} > 0$  ... this is true, so shade this region.

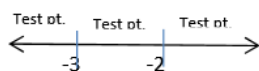
Practice:

1.  $\sqrt{3x+2} > 0$                       2.  $\sqrt{4x-7} > 0$                       3.  $\sqrt{x+8} > 0$

4.  $\sqrt{4x+8} < 0$                       5.  $\sqrt{\frac{27x}{3}} > 0$                       6.  $\sqrt{\frac{6}{18x}} < 0$

Now take this same process to make the number lines for polynomials

1.  $(x+3)(x+2) > 0$                       2.  $(x-4)(x+7) < 0$                       3.  $x^2+7x+12 > 0$



4.  $x^2-5x+6 < 0$                       6.  $x^2+4x-12 < 0$                       7.  $2x^2-7x-3 < 0$

Practice:

1.  $\sqrt{3x+2} > 0$

Step: → Set equal to zero ( $= 0$ )

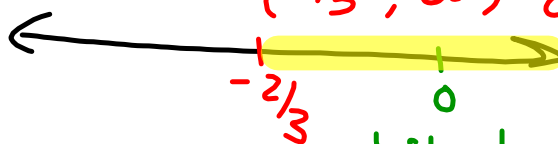
$$(\sqrt{3x+2})^2 = (0)^2$$

Solve for  $x$ .  $3x+2 = 0$

$$3x = -2$$

$$x = -2/3$$

$(-2/3, \infty)$  answer



test pt.  
 $x = 0$

$$\sqrt{3(0)+2} > 0$$

$$\sqrt{2} > 0$$

True, shade

1. 6 -1. -6  
2. 3 -2. -3

4.  $x^2 - 5x + 6 < 0$

\* Before I start

\* FACTOR

$(x - 2)(x - 3) < 0$

$x - 2 = 0$        $x - 3 = 0$

$x = 2$                        $x = 3$

( 2 , 3 )



Test pt.	2	Test pt.	3	test pt.
$x = 0$		$x = 2.5$		$x = 4$
$6 < 0$		$(2.5 - 2)(2.5 - 3) < 0$		$(4 - 2)(4 - 3) < 0$
False		$(.5)(-.5) < 0$		$(2)(1) < 0$
		$-.25 < 0$		$2 < 0$
		true		False

2.  $(x - 4)(x + 7) < 0$

① Set = 0

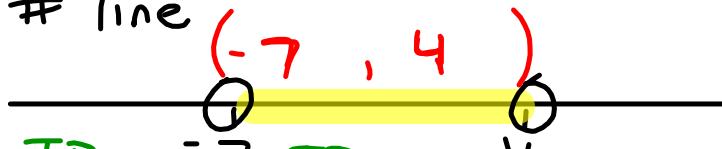
$(x - 4)(x + 7) = 0$

$x - 4 = 0$     $x + 7 = 0$

$x = 4$     $x = -7$

"<"  
open circles

② Plot on # line



③ test points

TP	-7	TP	4	TP	x = 5
x = -8		x = 0			
$(-8 - 4)(-8 + 7) < 0$		$(0 - 4)(0 + 7) < 0$		$(5 - 4)(5 + 7) < 0$	
$(-12)(-1) < 0$		$-4(7) < 0$		$(1)(12) < 0$	
$12 < 0$		$-28 < 0$		$12 < 0$	
False		True		False	



$$\sqrt{3x+2} \leq 4$$

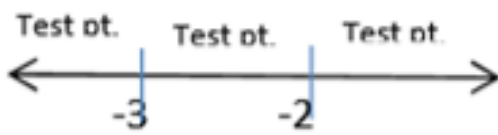
Needs to be 0.

$$(\sqrt{3x+2})^2 \leq (4)^2$$

$$3x+2 \leq 16$$

$$3x-14 \leq 0$$

1.  $(x + 3)(x + 2) > 0$



4.  $x^2 - 5x + 6 < 0$

1.  $\sqrt{3x + 2} > 0$

