

### Multiplication and Division of Radicals

- You can multiply/divide numbers outside of a radical together and you can multiply/divide numbers inside a radical.

\*Be sure to simplify after multiplying or dividing.

$$\sqrt{6} \cdot \sqrt{8} = \sqrt{6 \cdot 8} = \sqrt{48} \Rightarrow 4\sqrt{3}$$

$\sqrt{4 \cdot 2}$        $\sqrt{16 \cdot 3}$

$$\sqrt{6} \cdot 2\sqrt{2} = 2\sqrt{6 \cdot 2} = 2\sqrt{12}$$

$\sqrt{4 \cdot 3}$

$$\sqrt{10} \cdot 9\sqrt{6}$$

$$2 \cdot 2\sqrt{3} = 4\sqrt{3}$$

$$9\sqrt{60}$$

$$9 \cdot \sqrt{4 \cdot 15}$$

$$9 \cdot 2\sqrt{15}$$

$$18\sqrt{15}$$

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$$\sqrt{4 \cdot 2} \quad \sqrt{16 \cdot 3}$$

$$\sqrt{6} \cdot 2 \cdot \sqrt{2} = 2\sqrt{12} = 4\sqrt{3}$$

$$2 \cdot \sqrt{4 \cdot 3} = 2 \cdot 2\sqrt{3} = 4\sqrt{3}$$

$$\sqrt{10} \cdot 9\sqrt{6}$$

$$9\sqrt{60}$$

$$9\sqrt{4 \cdot 15}$$

$$9 \cdot 2\sqrt{15}$$

$$18\sqrt{15}$$

**Multiply Radical Expressions**

$$\begin{aligned}\sqrt{18} \cdot \sqrt{3} &= \sqrt{54} \\ &\quad \swarrow \searrow \\ &\quad \sqrt{9 \cdot 6} \\ &= 3\sqrt{6}\end{aligned}$$

$$\begin{aligned}\sqrt{3} \cdot \sqrt{21} &= \sqrt{63} \\ &\quad \swarrow \searrow \\ &\quad \sqrt{9 \cdot 7} \\ &\quad 3\sqrt{7}\end{aligned}$$

**Multiply Radical Expressions**

$$\sqrt{18} \cdot \sqrt{3} = \sqrt{54}$$
$$\sqrt{9 \cdot 6}$$
$$3\sqrt{6}$$

$$\sqrt{3} \cdot \sqrt{21} = \sqrt{63}$$
$$\sqrt{9 \cdot 7}$$
$$3\sqrt{7}$$

$$(x+1)(x-1) = x^2 - 1$$

- FOIL
- Distribute

$$(x+3)(x-3)$$

$$x^2 - 3x + 3x - 9$$

$$x^2 - 9$$

$$(x+5)(x-5)$$

$$x^2 - 25$$

$$(x + \sqrt{3})(x - \sqrt{3})$$

$$x^2 - x\sqrt{3} + x\sqrt{3} + (-\sqrt{3})(\sqrt{3})$$

$$x^2 - 3$$

$$(x - \sqrt{2})(x + \sqrt{2})$$

$$(-\sqrt{2})(+\sqrt{2})$$

$$x^2 - 2$$

$$(3x - \sqrt{5})(3x + \sqrt{5})$$

$$9x^2 - 5$$

What is

$$(x+1)(x-1)$$

$$x^2 - 1$$

FOIL  
or  
Distribute

$$(x+4)(x-4)$$

$$x^2 - 16$$

$$(x+\sqrt{3})(x-\sqrt{3})$$

$$x^2 + (\sqrt{3})(-\sqrt{3})$$

$$-\sqrt{3} \cdot \sqrt{3} = -3$$

$$x^2 - 3$$

$$(x-\sqrt{2})(x+\sqrt{2})$$

$$x^2 - 2$$

$$(3x-\sqrt{5})(3x+\sqrt{5})$$

$$9x^2 - 5$$

$$(x-\sqrt{7})(x+\sqrt{7})$$

$$x^2 - 7$$

## How to Divide "Rationalize the Denominator"

\*Use Multiplication

Case 1

$$\frac{5}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$$

What is the  
difference?

Case 2

$$\frac{5}{3+\sqrt{21}} \cdot \frac{(3-\sqrt{21})}{(3-\sqrt{21})}$$

$$\frac{15-5\sqrt{21}}{-12}$$

# How to Divide "Rationalize the Denominator"

\*Use Multiplication

Case 1

$$\frac{5}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}}$$

$$\frac{5\sqrt{21}}{21}$$

What is the difference?

Case 2

$$\frac{5}{3+\sqrt{21}}$$

$$\frac{5(3-\sqrt{21})}{(3+\sqrt{21})(3-\sqrt{21})} = \frac{15-5\sqrt{21}}{-12}$$

$9 - 3\sqrt{21} + 3\sqrt{21} - 21$   
 $9 - 21$   
 $-12$

Rationalizing the Denominator of Radicals

$$\frac{7}{\sqrt{75}} = \frac{7\sqrt{75}}{75} = \frac{7 \cdot 5\sqrt{3}}{75} = \frac{35\sqrt{3}}{75}$$

The handwritten work shows the following steps and annotations:

- Initial fraction:  $\frac{7}{\sqrt{75}}$
- Step 1:  $\frac{7\sqrt{75}}{75}$  (The  $7\sqrt{75}$  is highlighted in yellow).
- Step 2:  $\frac{7 \cdot 5\sqrt{3}}{75}$  (The  $7 \cdot 5\sqrt{3}$  is highlighted in yellow).
- Step 3:  $\frac{35\sqrt{3}}{75}$  (The final result is circled in blue).
- A blue bracket groups the intermediate steps, with a vertical line between  $\sqrt{75}$  and  $\sqrt{25 \cdot 3}$ , and  $5\sqrt{3}$  is highlighted in yellow.

$$\frac{3}{\sqrt{40}} \cdot \frac{(\sqrt{40})}{(\sqrt{40})} = \frac{3\sqrt{40}}{40} = \frac{6\sqrt{10}}{40}$$

The handwritten work shows the following steps and annotations:

- Initial fraction:  $\frac{3}{\sqrt{40}}$
- Step 1:  $\frac{3\sqrt{40}}{40}$  (The  $3\sqrt{40}$  is highlighted in blue).
- Step 2:  $\frac{6\sqrt{10}}{40}$  (The final result is circled in green).
- A green arrow points from the  $3\sqrt{40}$  to the  $6\sqrt{10}$ , with a circled 4 and an arrow pointing to the 10, indicating the simplification of  $3\sqrt{40} = 3 \cdot 2\sqrt{10} = 6\sqrt{10}$ .

$$\frac{3}{(2-\sqrt{5})(2+\sqrt{5})} = \frac{3(2+\sqrt{5})}{4-5} = \frac{(6+3\sqrt{5})}{-1}$$

$$= -\left(\frac{6+3\sqrt{5}}{1}\right)$$

$$= -(6+3\sqrt{5})$$

$$\frac{4x}{x-\sqrt{3}} \cdot \frac{(x+\sqrt{3})}{(x+\sqrt{3})} = \frac{4x^2 + 4x\sqrt{3}}{x^2 - 3} = -6 - 3\sqrt{5}$$

$$\frac{2x+1}{x+\sqrt{3}} \cdot \frac{(x-\sqrt{3})}{(x-\sqrt{3})} = \frac{2x^2 - 2x\sqrt{3} + x - \sqrt{3}}{x^2 - 3}$$

$$\frac{6}{(3-\sqrt{2})(3+\sqrt{2})} \cdot \frac{(3+\sqrt{2})}{(3+\sqrt{2})} = \frac{6(3+\sqrt{2})}{9-2} = \frac{6(3+\sqrt{2})}{7}$$

$$= \frac{18+6\sqrt{2}}{7}$$

$$\frac{2x}{(x+\sqrt{3})(x-\sqrt{3})} \cdot \frac{(x-\sqrt{3})}{(x-\sqrt{3})} = \frac{2x(x-\sqrt{3})}{(x^2-3)}$$

$$= \frac{2x^2 - 2x\sqrt{3}}{x^2-3}$$

$$\frac{(x+1)(x+\sqrt{5})}{(x-\sqrt{5})(x+\sqrt{5})} = \frac{(x+1)(x+\sqrt{5})}{(x^2-5)}$$

$$\frac{x^2 + x\sqrt{5} + x + \sqrt{5}}{(x^2-5)}$$