


What is a Rational Function?   $y = \frac{1}{x}$  Fraction

horizontal asymptote  
vertical asymptote

What is the restriction of rational functions?  $y = \frac{1}{x}$   $x \neq 0$

Does this relate to the Domain or the Range? Domain

Why?  $x$ -values

The image contains several handwritten mathematical expressions and a note:

- $\frac{1}{0} = ?$  (circled in green, with a red arrow pointing to the denominator)
- $\frac{4}{2} = 2$  (circled in green, with a red arrow pointing to the denominator)
- $\frac{10}{2} = 5$  (circled in green, with a red arrow pointing to the denominator)
- $\frac{2}{3} = 5$  (circled in green, with a red arrow pointing to the denominator)
- $\frac{10}{3} = 6$  (circled in green, with a red arrow pointing to the denominator)
- Text: can't happen (written in red)

What are restrictions?  $x \neq 0$  asymptote

This is when you set the denominator = 0

How do you write your answers?

$\mathbb{R}; x \neq -, -, \dots$

"(all reals) except "

Let's Practice

1) 
$$\frac{p^2 - 3p - 10}{p - 5}$$



2) 
$$\frac{m + 9}{5m^2 + 45m}$$

3) 
$$\frac{10x + 10}{10x + 15}$$

4) 
$$\frac{n^2 - 5n - 6}{36 - n^2}$$

5) 
$$\frac{x^3 + 2x^2 - 3x}{2x - 2}$$

6) 
$$\frac{2r^3 + 2r^2 - 112r}{3r^3 - 27r^2 + 42r}$$

7) 
$$\frac{24n^3 - 8n^2 - 16n}{56n^3 - 40n^2 - 16n}$$

8) 
$$\frac{2m^2 - 12m - 80}{5m + 20}$$

$$1) \frac{p^2 - 3p - 10}{p - 5}$$

$$p \neq 5$$

work :

$$\begin{array}{r} p - 5 \neq 0 \\ + 5 \\ \hline p \neq 5 \end{array}$$

$$2) \frac{m + 9}{5m^2 + 45m}$$

$$5m(m + 9) \neq 0$$

$$5m \neq 0 \quad m \neq 0$$

$$m + 9 \neq 0$$

$$\begin{array}{r} -9 \\ \hline m \neq -9 \end{array}$$

GCF

$$\begin{array}{l} 5m^2 + 45m = 0 \\ 5m \quad 5m \quad 9 \end{array}$$

$$\begin{array}{l} m + 9 = 0 \\ m \neq -9 \end{array}$$

$$3) \frac{10x + 10}{10x + 15}$$

$$10x + 15 = 0$$

-15

$$\frac{10x}{10} = \frac{-15}{10}$$

$$x = \frac{-15}{10} = \frac{-3}{2}$$

$$x \neq \frac{-3}{2}$$

$$4) \frac{n^2 - 5n - 6}{36 - n^2} \quad \begin{array}{l} n^2 - 36 = (n-6)(n+6) \\ = (6-n)(6+n) \end{array}$$

$$36 - n^2 = 0$$

$$-36 \quad -36$$

$$\sqrt{n^2} = \sqrt{36}$$

$$n \neq \pm 6$$

$$5) \frac{x^3 + 2x^2 - 3x}{2x - 2}$$

$$2x - 2 \neq 0$$

$$\frac{2x}{2} \neq \frac{2}{2}$$

$$x \neq 1$$

$$6) \frac{2r^3 + 2r^2 - 112r}{3r^3 - 27r^2 + 42r}$$

$$3r(r^2 - 9r + 14)$$

$$3r(r-2)(r-7)$$

$$r \neq 0 \quad r \neq 2 \quad r \neq 7$$



7)  $\frac{24n^3 - 8n^2 - 16n}{56n^3 - 40n^2 - 16n}$

8)  $\frac{2m^2 - 12m - 80}{5m + 20}$

Find:

① Restrictions

② # 1-8

FACTOR

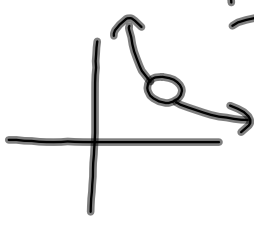
$\frac{\text{TOP}}{\text{BOTTOM}}$

1:00

# Asymptotes

Vertical : Restrictions  
denominator = 0

holes :  $\frac{\text{TOP}}{\text{Bottom}}$  } cancel out terms



$$\frac{\cancel{(x+1)}(x+3)}{2x\cancel{(x+1)}}$$

$x+1=0$   
 $x \neq -1$  Hole

$2x=0$   
 $x \neq 0$   
 Vertical asymptote

Horizontal : Rules

- ①
- ②
- ③

1) 
$$\frac{p^2 - 3p - 10}{p - 5}$$

$$p + 2; \{5\}$$

2) 
$$\frac{m + 9}{5m^2 + 45m}$$

$$\frac{1}{5m}; \{0, -9\}$$

3) 
$$\frac{10x + 10}{10x + 15}$$

$$\frac{2(x + 1)}{2x + 3}; \left\{ -\frac{3}{2} \right\}$$

4) 
$$\frac{n^2 - 5n - 6}{36 - n^2}$$

$$-\frac{(n + 1)}{6 + n}; \{6, -6\}$$

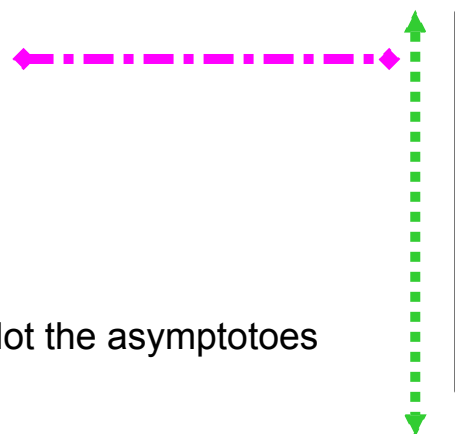
$$5) \frac{x^3 + 2x^2 - 3x}{2x - 2}$$
$$\frac{x(x+3)}{2}; \{1\}$$

$$6) \frac{2r^3 + 2r^2 - 112r}{3r^3 - 27r^2 + 42r}$$
$$\frac{2(r+8)}{3(r-2)}; \{0, 7, 2\}$$

$$7) \frac{24n^3 - 8n^2 - 16n}{56n^3 - 40n^2 - 16n}$$
$$\frac{3n+2}{7n+2}; \left\{0, 1, -\frac{2}{7}\right\}$$

$$8) \frac{2m^2 - 12m - 80}{5m + 20}$$
$$\frac{2(m-10)}{5}; \{-4\}$$

What happens when you graph?



Plot the asymptotes

Insert the y-axis

Sketch the graph of the function

## Graphing a Rational Function

Vertical Asymptotes

Horizontal Asymptotes



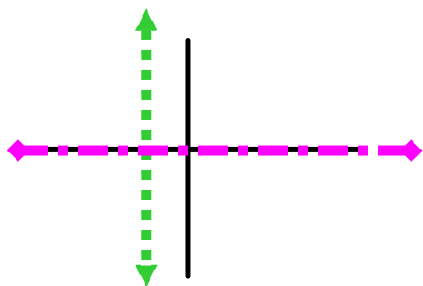
## Graphing Rational Functions

What is the end behavior?

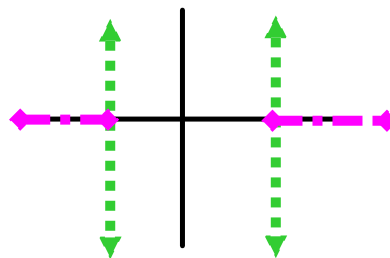
What is the highest degree of the rational function?

How many vertical asymptotes are there?

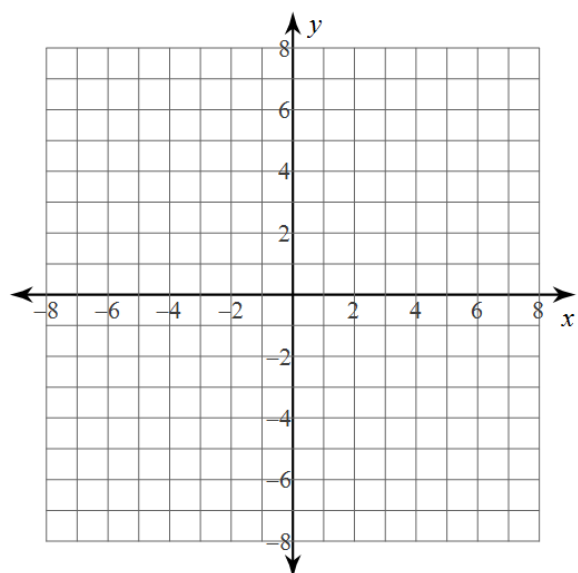
1 vertical asymptotes: \_\_\_\_\_



2 vertical asymptotes: \_\_\_\_\_

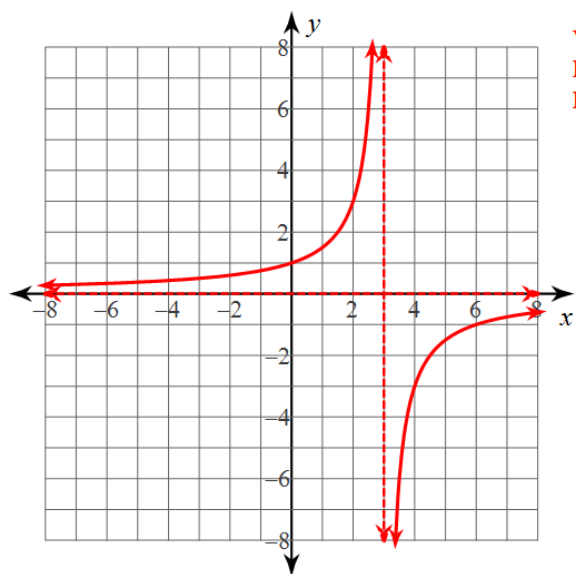


1)  $f(x) = -\frac{3}{x-3}$



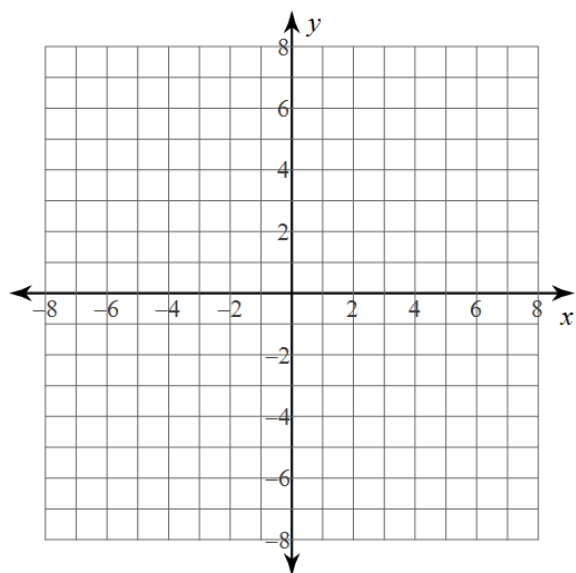


1)  $f(x) = -\frac{3}{x-3}$

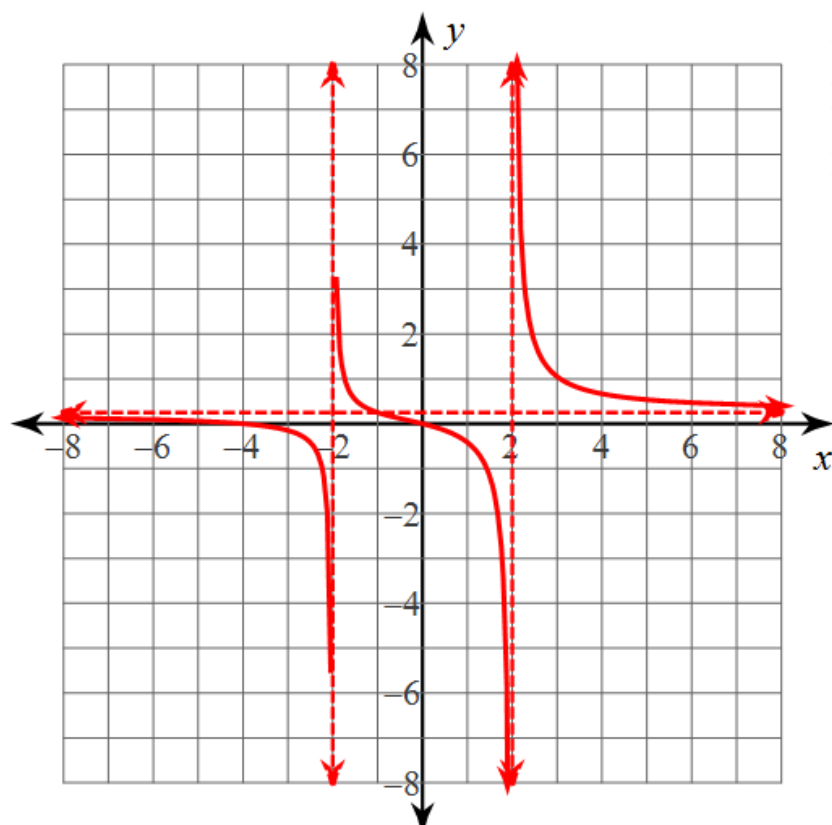


Vertical Asym.:  $x = 3$   
Holes: None  
Horz. Asym.:  $y = 0$

$$2) f(x) = \frac{x^2 + 4x}{4x^2 - 16}$$



$$2) f(x) = \frac{x^2 + 4x}{4x^2 - 16}$$

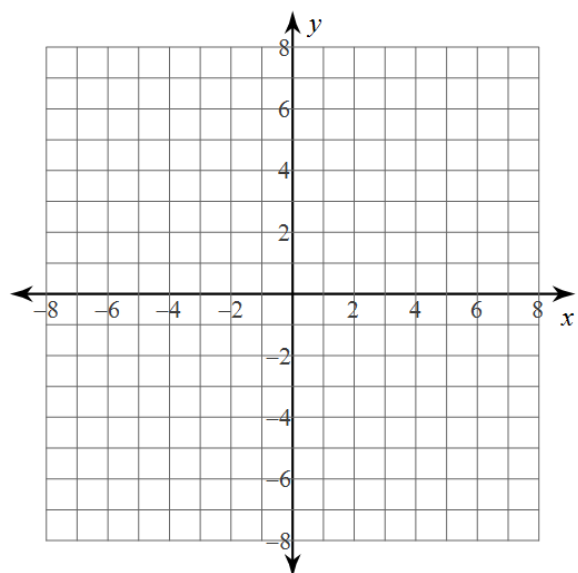


Vertical Asym.:  $x = 2, x = -2$

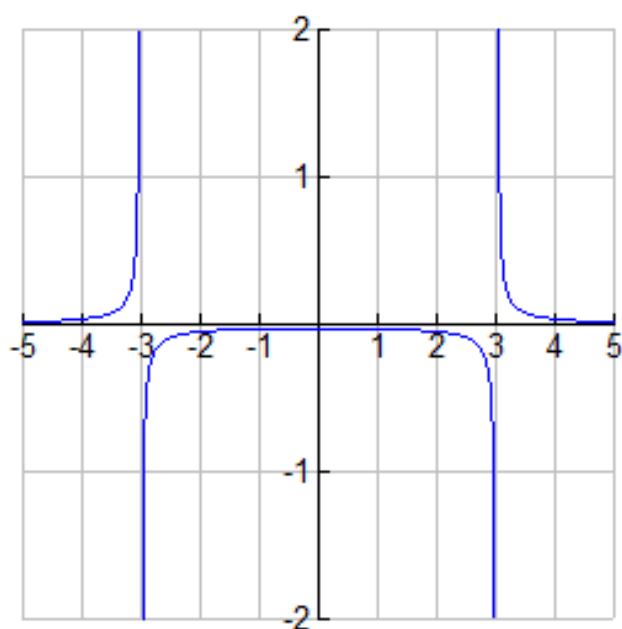
Holes: None

Horz. Asym.:  $y = \frac{1}{4}$

3)  $f(x) = \frac{1}{4x^2 - 36}$



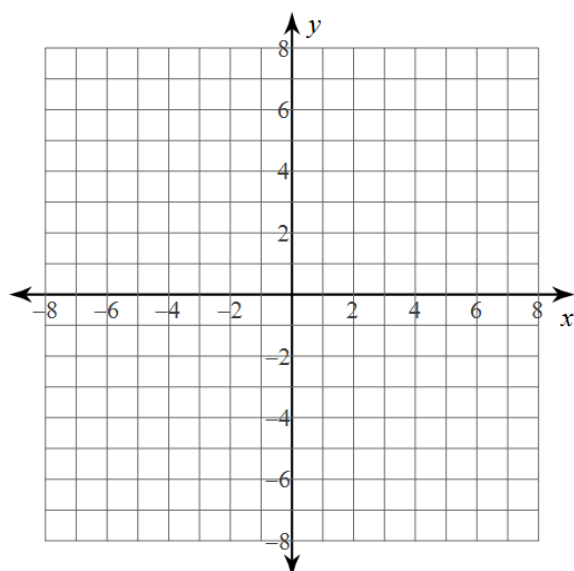
$$3) f(x) = \frac{1}{4x^2 - 36}$$



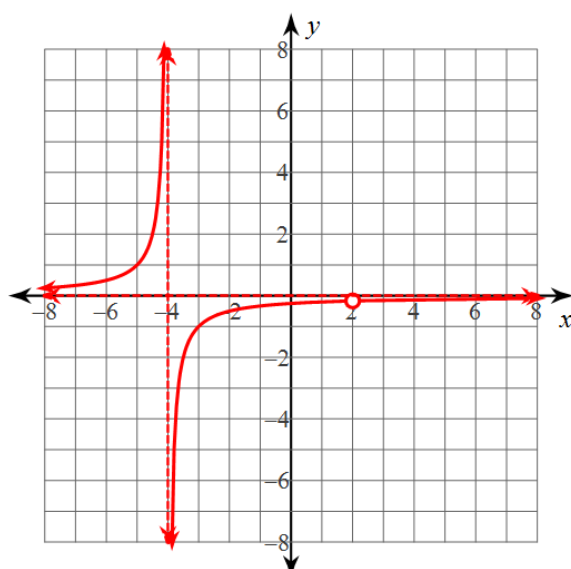
Vertical Asym.:  $x = 3, x = -3$   
Holes: None  
Horz. Asym.:  $y = 0$

-2.

$$4) f(x) = \frac{x - 2}{-x^2 - 2x + 8}$$

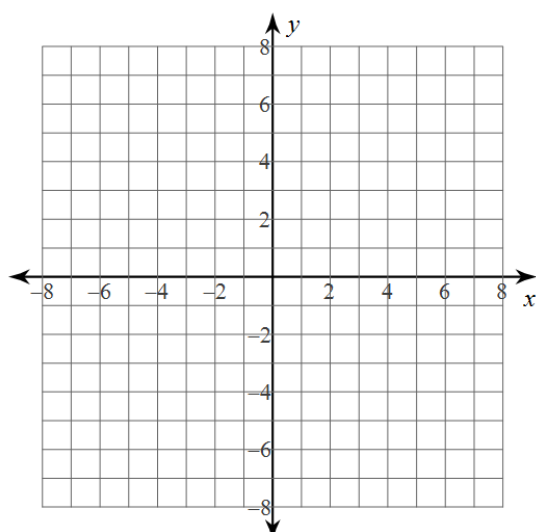


$$4) f(x) = \frac{x - 2}{-x^2 - 2x + 8}$$

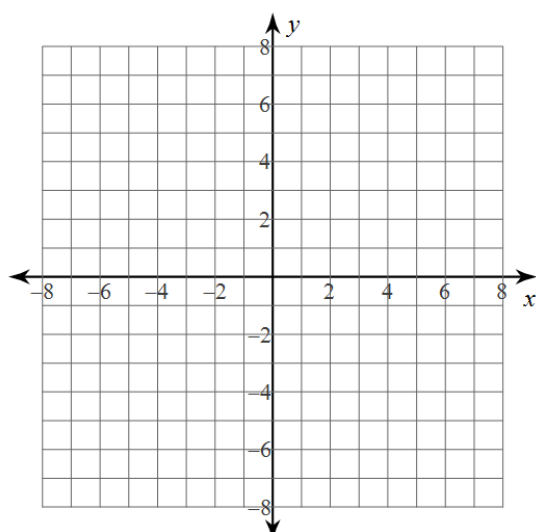


Vertical Asym.:  $x = -4$   
Holes:  $x = 2$   
Horz. Asym.:  $y = 0$

1)  $f(x) = -\frac{x}{x^2 - 4x}$

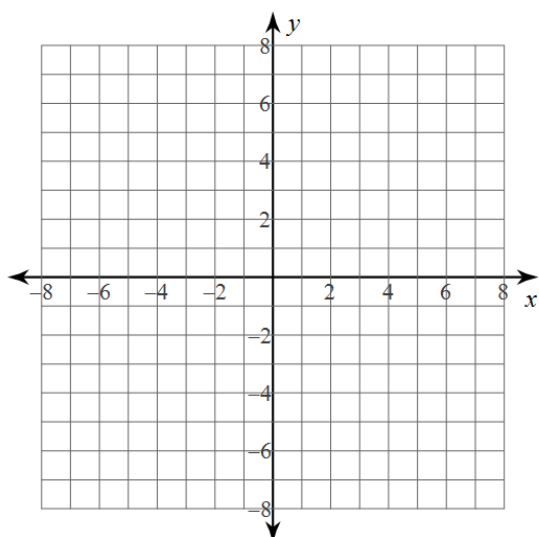


2)  $f(x) = -\frac{3}{x^2 - 2x - 3}$





3)  $f(x) = \frac{x^2 + 4x}{x^2 - 2x - 3}$



4)  $f(x) = \frac{-2x + 4}{x + 2}$

