What is a Rational Function? $y = \frac{1}{x}$ a fraction

Restrictions denominator = 0

What is the restriction of rational functions? undefined

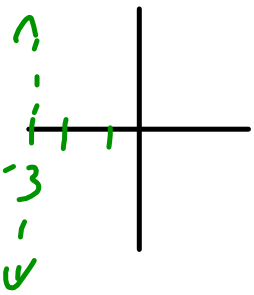
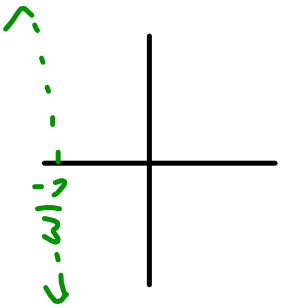
$y = \frac{1}{x}$ ← restriction goes with "x"

Does this relate to the Domain or the Range? Domain

x values y values

Why? Domain are x-values and the restriction are what $x \neq$

Restrictions & Graphing

	Restriction denom = 0	asymptote	on the graph
$y = \frac{1}{x+3}$	$x+3=0$ $x \neq -3$	$x+3=0$ $x=-3$	
$y = \frac{1}{3x+7}$	$3x+7=0$ $3x=-7$ $x \neq -\frac{7}{3}$	$3x+7=0$ $x=-\frac{7}{3}$	

Practice

① $y = \frac{1}{x+3}$

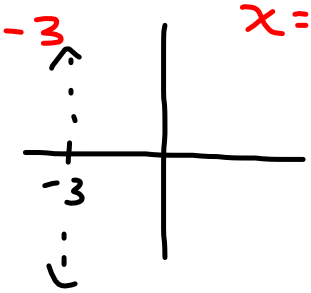
Restrictions ⇒ vertical asymptote

$x+3=0$

$x \neq -3$

$x = -3$

graph

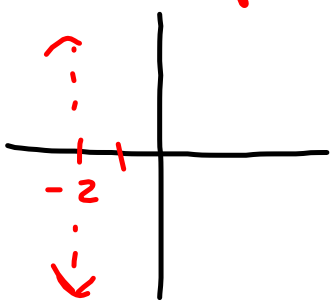


② $y = \frac{1}{x+2} - 4$

$x+2 \neq 0$

$x \neq -2$

Graph $x = -2$



③ $y = \frac{1}{2x+5} + 2$

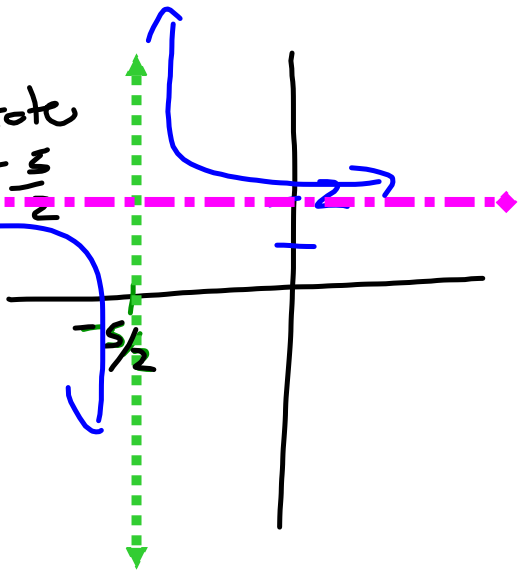
Restriction ⇒ vertical asymptote

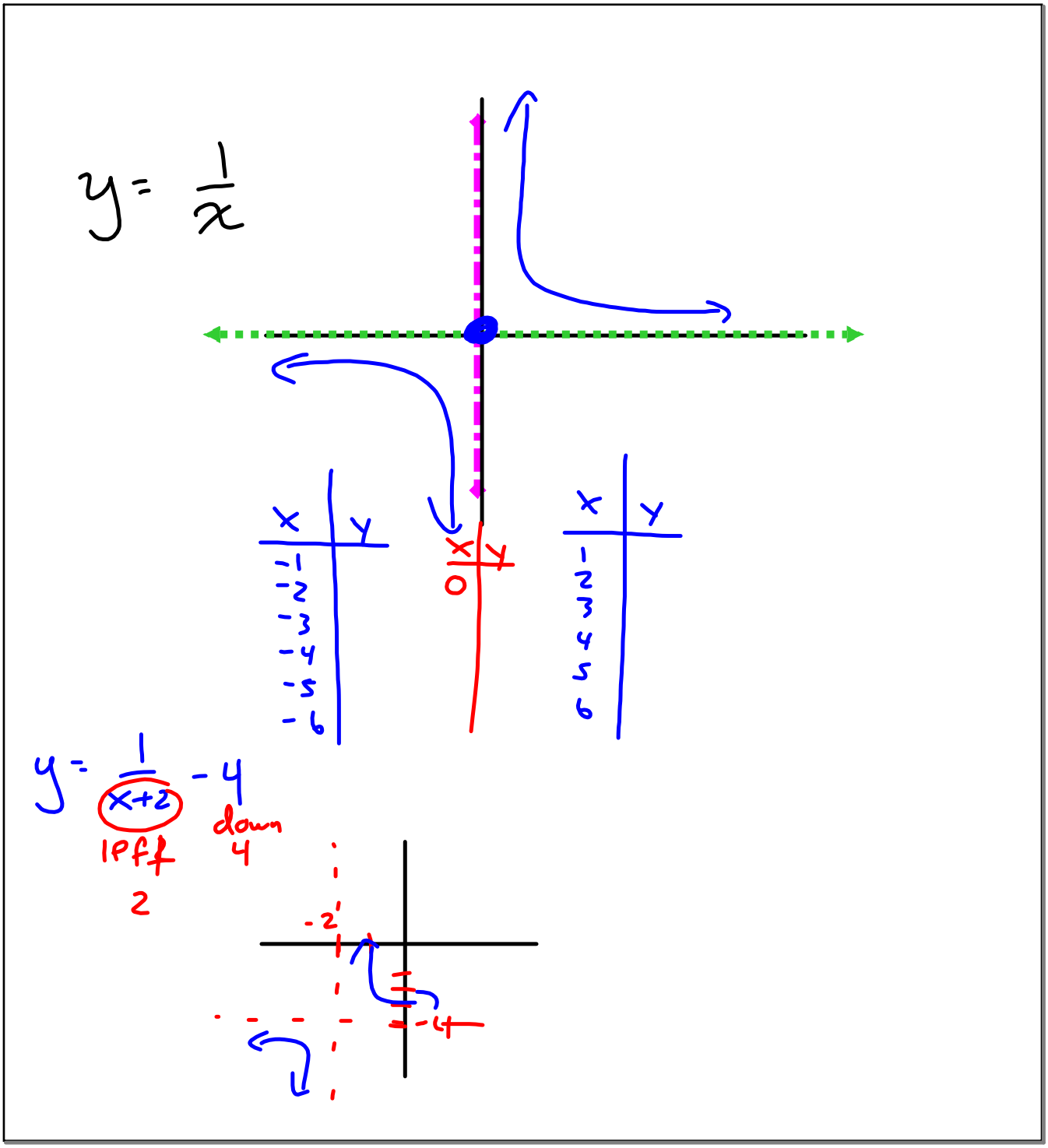
$2x+5=0$

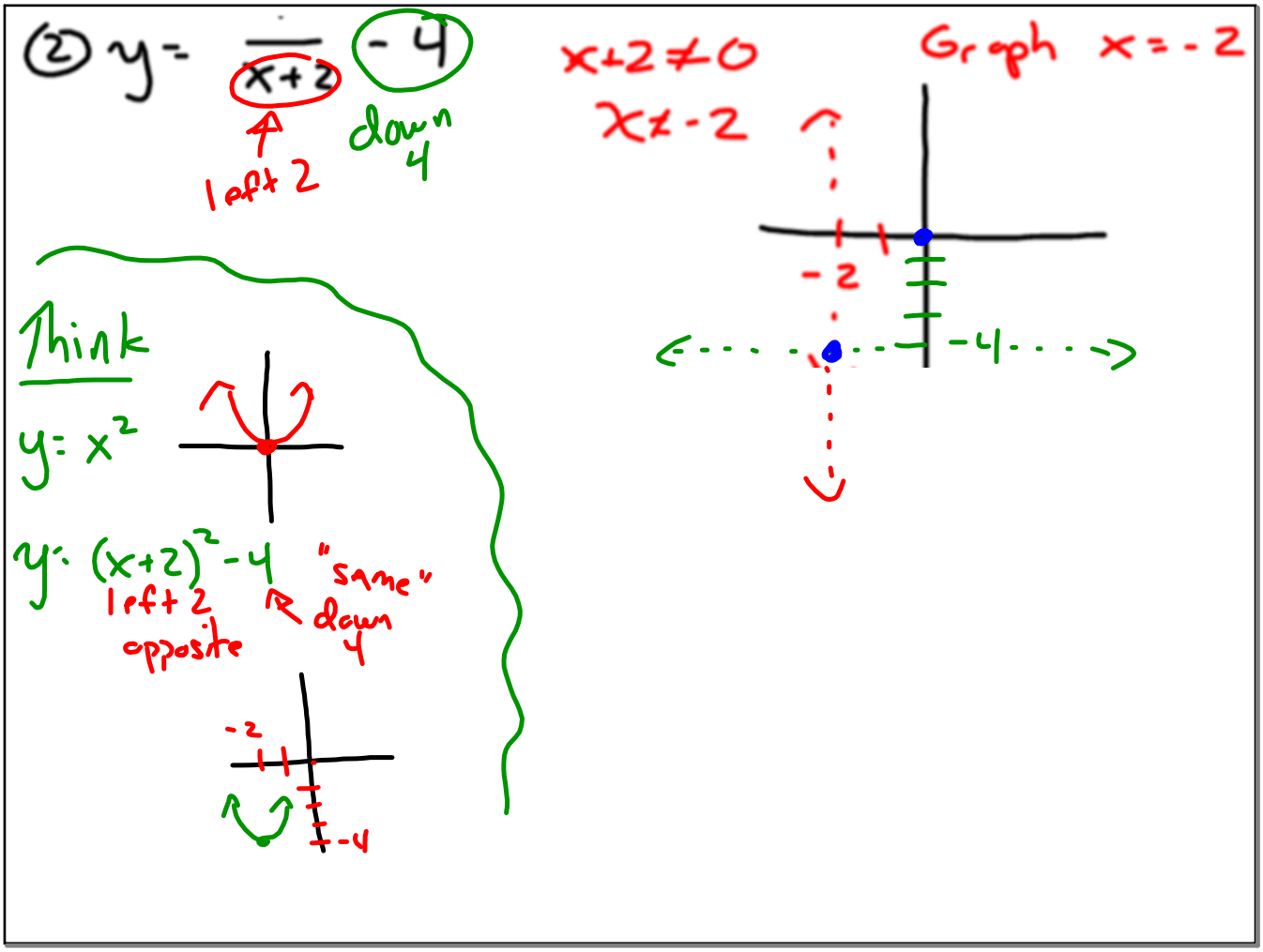
$2x = -5$

$x \neq \frac{-5}{2}$

$x = \frac{-5}{2}$




$y = \frac{1}{x}$
x y
-1
-2
-3
-4
-5
-6
x y
0
x y
1
2
3
4
5
6
$y = \frac{1}{x+2} - 4$
left 2
down 4
-2
-4


② y = 1/(x+2) - 4
left 2
down 4
x+2 ≠ 0
x ≠ -2
Graph x = -2
-2
-4
Think
y = x²
y = (x+2)² - 4
left 2
opposite
"same"
down 4
-2
-4

Why are there asymptotes?

restriction $x \neq$ → makes vertical asymptote

$y \neq$ horizontal asymptote } Rules

highest degree
- bottom $y = 0$
- same $y =$ coefficients
- top "slant"

What makes a vertical asymptote?

* undefined values

1) $\frac{p^2 - 3p - 10}{p - 5}$

2) $\frac{m + 9}{5m^2 + 45m}$

3) $\frac{10x + 10}{10x + 15}$

4) $\frac{n^2 - 5n - 6}{36 - n^2}$

5) $\frac{x^3 + 2x^2 - 3x}{2x - 2}$

6) $\frac{2r^3 + 2r^2 - 112r}{3r^3 - 27r^2 + 42r}$

7) $\frac{24n^3 - 8n^2 - 16n}{56n^3 - 40n^2 - 16n}$

8) $\frac{2m^2 - 12m - 80}{5m + 20}$

① simplify

② denom = 0

③ look at highest degree

④ cancel out = 0
* hole *

⑤ graph

Feb 11-2:54 PM

What is a Rational Function? $y = \frac{1}{x}$ why? Fractions

What is the restriction of rational functions? $x \neq \square$ denom = 0
• undefined

Does this relate to the Domain or the Range? Domain

all "x" values all "y" values

Why? restriction $x \neq$ is the domain

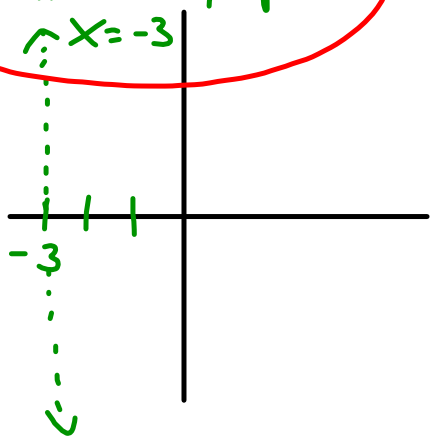
$y = \frac{1}{x+3}$

Restriction

$x \neq -3$

Vertical asymptote

$x = -3$



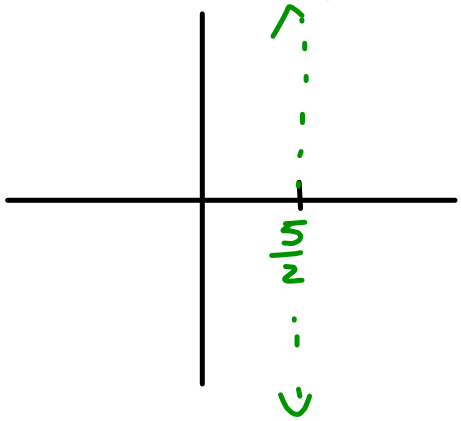
$y = \frac{1}{2x-5}$

restriction

$x \neq \frac{5}{2}$

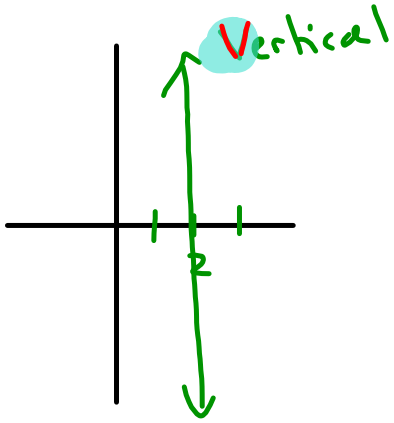
Vertical asymptote

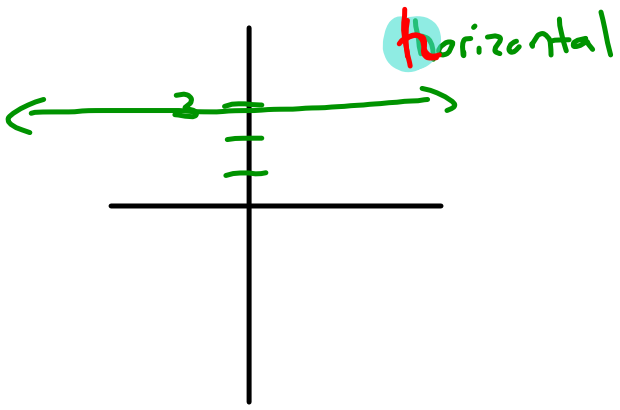
$x = 5/2$



$x = 2$ $y = 3$

What type of lines?





Why are there asymptotes?

restriction $x \neq$ → makes vertical asymptote

$y \neq$ horizontal asymptote } Rules

highest degree
- bottom $y = 0$
- same $y =$ coefficients
- top "slant"

What makes a vertical asymptote?

* undefined values

Why are there asymptotes?

Vertical

denom = 0

where x values are undefined

horizontal => Rules

highest degree on bottom $y = 0$

highest degree on top slant

same degree y= coefficients

What makes a vertical asymptote?

$$y = \frac{(x+3)}{(x+3)(x+2)}$$

1. simplify
2. denom = 0 — Vertical asymptotes
3. cancel out = 0 — holes
4. degree — horizontal asymptotes
5. graph

$(x+3)=0$ $(x+2)=0$

~~$x=-3$~~ $x=-2$

$$\frac{\cancel{(x+3)}}{\cancel{(x+3)}(x+2)}$$

$x+3=0$

$x=-3$ Hole

$$y = \frac{x^2+7x+12}{x^2+5x+4}$$

① simplify → $y = \frac{(x+3)(x+4)}{(x+1)(x+4)}$

② denom = 0 Vertical asymptotes $x+1=0$ $x=-1$ VA

③ cancel out = 0 holes $\frac{(x+3)(\cancel{x+4})}{(x+1)(\cancel{x+4})}$ $x+4=0$ $x=-4$ Hole

④ degree horizontal asymptotes

⑤ graph

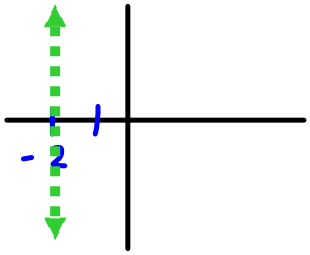
Restrictions => VA =>

"VA" vertical asymptote

① $y = \frac{1}{x+2}$

Restriction
denom ≠ 0
$x+2 \neq 0$
$x \neq -2$

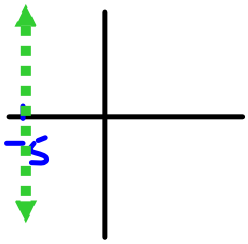
VA
denom = 0
$x+2=0$
$x = -2$



② $y = \frac{1}{x+5}$

$x+5 \neq 0$
$x \neq -5$

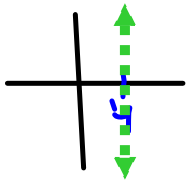
$x = -5$



③ $y = \frac{1}{x-4}$

$x-4 \neq 0$
$x \neq 4$

$x = 4$

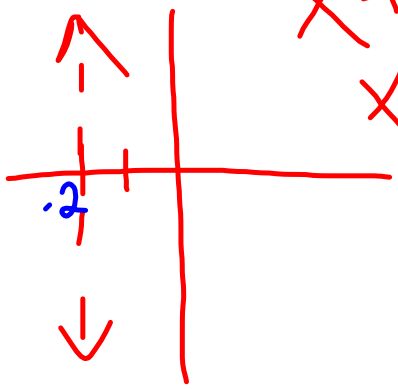


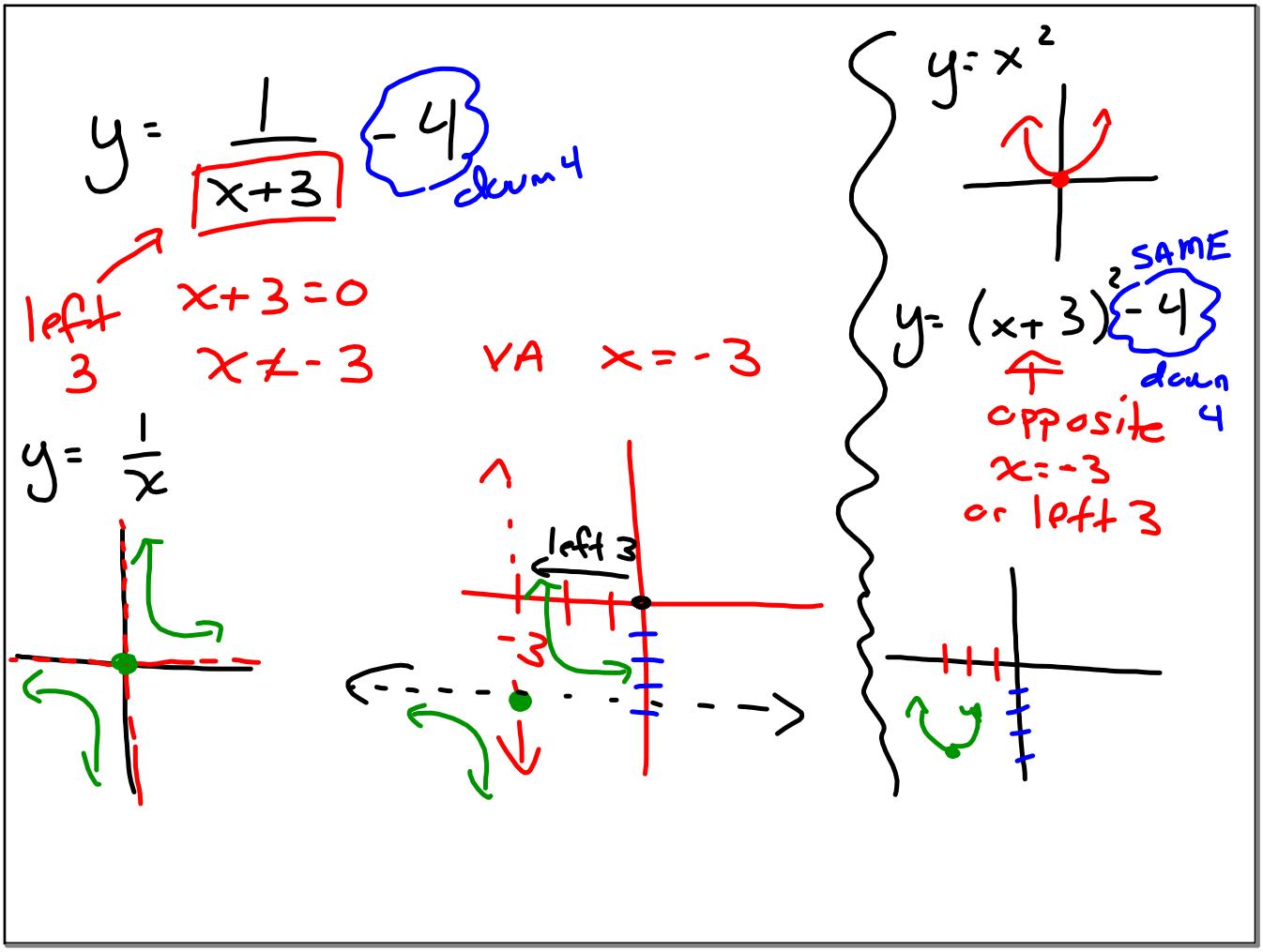
Oct 14-8:31 AM

$y = \frac{1}{x+2}$ $x \neq -2$ $x = -2$ VA

$x+2=0$

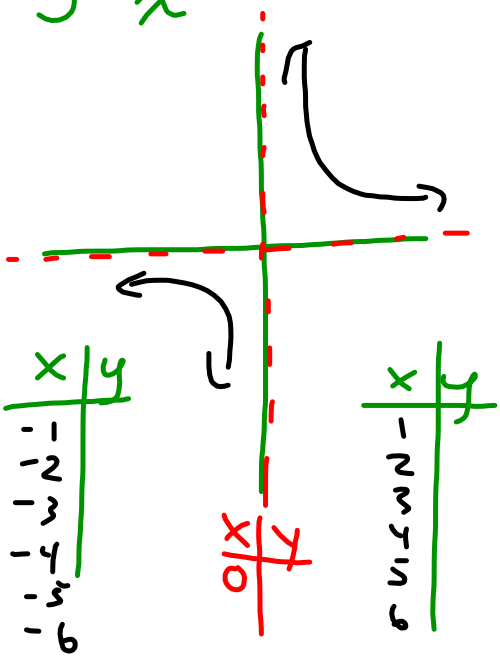
$x \neq -2$ $VA = -2$




y = 1/(x+3) - 4
down 4
left 3
x+3=0
x≠-3
VA x=-3
y = 1/x
left 3
-3
y = x²
y = (x+3)² - 4
SAME
down 4
opposite
x=-3
or left 3

Why?

$y = \frac{1}{x}$



x	y
-1	
-2	
-3	
-4	
-5	
-6	

x	y
0	

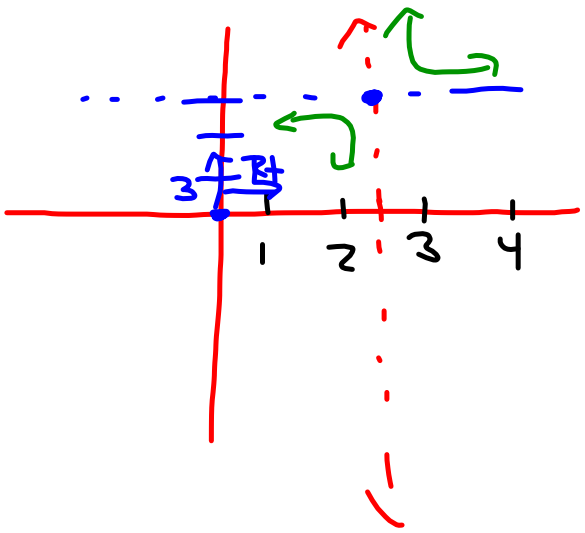
x	y
1	
2	
3	
4	
5	
6	

$y = \frac{1}{2x-5} + 3$

$2x - 5 = 0$

$2x = 5$

$x = 5/2$



$y = \frac{1}{3x-6} + 4$

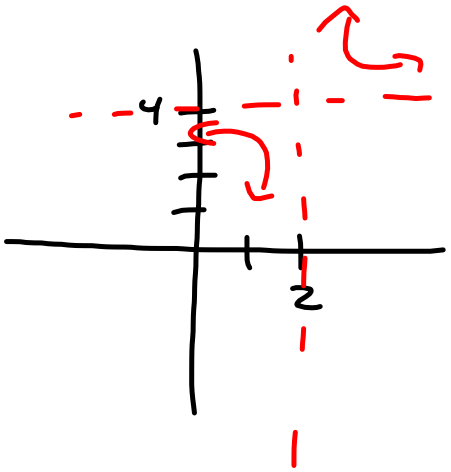
VA

HA

$3x - 6 = 0$

$3x = 6$

$x = 2$



$y = \frac{x(x+2)}{(x+2)(x+3)}$

denom = 0

$x+2=0$ $x+3=0$

~~$x=-2$~~ $x=-3$

cancel out

$y = \frac{x(x+2)}{(x+2)(x+3)}$

$x+2=0$ "Hole"

$x=-2$

$y = \frac{x^2+7x+12}{x^2+5x+4}$

① simplify

② denom = 0 Vertical asymptotes

③ cancel out = 0 "Holes"

④ horizontal asymptotes "Rules"

⑤ graph

1) $\frac{p^2 - 3p - 10}{p - 5}$

2) $\frac{m + 9}{5m^2 + 45m}$

3) $\frac{10x + 10}{10x + 15}$

4) $\frac{n^2 - 5n - 6}{36 - n^2}$

5) $\frac{x^3 + 2x^2 - 3x}{2x - 2}$

6) $\frac{2r^3 + 2r^2 - 112r}{3r^3 - 27r^2 + 42r}$

7) $\frac{24n^3 - 8n^2 - 16n}{56n^3 - 40n^2 - 16n}$

8) $\frac{2m^2 - 12m - 80}{5m + 20}$

① simplify

② denom = 0

③ look at highest degree

④ cancel out = 0
* hole *

⑤ graph

Feb 11-2:54 PM

1) $\frac{p^2 - 3p - 10}{p - 5}$ $\Rightarrow$ simplify: $\frac{(p-5)(p+2)}{(p-5)=0}$

VA: $p = 5$

Hole @ $p - 5 = 0$, $p = 5$

3) $\frac{10x + 10}{10x + 15}$ $\Rightarrow$ $\frac{10(x+1)}{5(2x+3)}$

VA: $5(2x+3) = 0$

1) $\frac{p^2 - 3p - 10}{p - 5}$

2) $\frac{m + 9}{5m^2 + 45m}$

3) $\frac{10x + 10}{10x + 15}$

4) $\frac{n^2 - 5n - 6}{36 - n^2}$

5) $\frac{x^3 + 2x^2 - 3x}{2x - 2}$

6) $\frac{2r^3 + 2r^2 - 112r}{3r^3 - 27r^2 + 42r}$

7) $\frac{24n^3 - 8n^2 - 16n}{56n^3 - 40n^2 - 16n}$

8) $\frac{2m^2 - 12m - 80}{5m + 20}$

Feb 11-2:54 PM

1) $\frac{p^2 - 3p - 10}{p - 5}$

$p \neq 5$

Work:

$p - 5 \neq 0$

$+5 \quad +5$

$p \neq 5$

2) $\frac{m + 9}{5m^2 + 45m}$

$5m(m+9) \neq 0$

$5m \neq 0 \qquad m \neq 0$

$m + 9 \neq 0$

$-9 \quad -9$

$m \neq -9$

GCF

$5m^2 + 45m = 0$

$5m \quad 5m$

$m + 9 = 0$

$m \neq -9$

3) $\frac{10x+10}{10x+15}$

$10x+15=0$

-15

$\frac{10x}{10}=\frac{-15}{10}$

$x=\frac{-15}{10}=\frac{-3}{2}$

$x\neq\frac{-3}{2}$

4) $\frac{n^2-5n-6}{36-n^2}$ $n^2-36=(n-6)(n+6)$

$=(6-n)(6+n)$

$36-n^2=0$

$-36 \quad -36$

$\sqrt{\pm n^2}=\sqrt{\pm 36}$

$n\neq\pm 6$

Feb 11-2:54 PM

5) $\frac{x^3+2x^2-3x}{2x-2}$

$2x-2\neq 0$

$\frac{2x}{2}\neq\frac{2}{2}$

$x\neq 1$

6) $\frac{2r^3+2r^2-112r}{3r^3-27r^2+42r}$

$3r(r^2-9r+14)$

$3r(r-2)(r-7)$

$r\neq 0 \quad r\neq 2 \quad r\neq 7$

Feb 11-2:54 PM

7) $\frac{24n^3 - 8n^2 - 16n}{56n^3 - 40n^2 - 16n}$

8) $\frac{2m^2 - 12m - 80}{5m + 20}$

Find:

① Restrictions

② #1-8

FACTOR TOP / BOTTOM

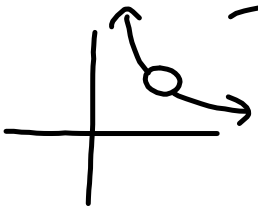
1:00

Asymptotes

Vertical : Restrictions

denominator = 0

holes : $\frac{\text{Top}}{\text{Bottom}}$ } cancel out term



$$\frac{(x+1)(x+3)}{2x(x+1)}$$

$x+1=0$
$x\neq -1$ Hole

Vertical asymptot

$2x=0$
$x\neq 0$

Horizontal : Rules

1.
2.
3.

1) $\frac{p^2-3p-10}{p-5}$ $p+2\ ;\ \{5\}$	2) $\frac{m+9}{5m^2+45m}$ $\frac{1}{5m}\ ;\ \{0,-9\}$
3) $\frac{10x+10}{10x+15}$ $\frac{2(x+1)}{2x+3}\ ;\ \left\{-\frac{3}{2}\right\}$	4) $\frac{n^2-5n-6}{36-n^2}$ $-\frac{(n+1)}{6+n}\ ;\ \{6,-6\}$

Feb 11-2:55 PM

5) $\dfrac{x^3+2x^2-3x}{2x-2}$

$\dfrac{x(x+3)}{2}$; $\{1\}$

6) $\dfrac{2r^3+2r^2-112r}{3r^3-27r^2+42r}$

$\dfrac{2(r+8)}{3(r-2)}$; $\{0, 7, 2\}$

7) $\dfrac{24n^3-8n^2-16n}{56n^3-40n^2-16n}$

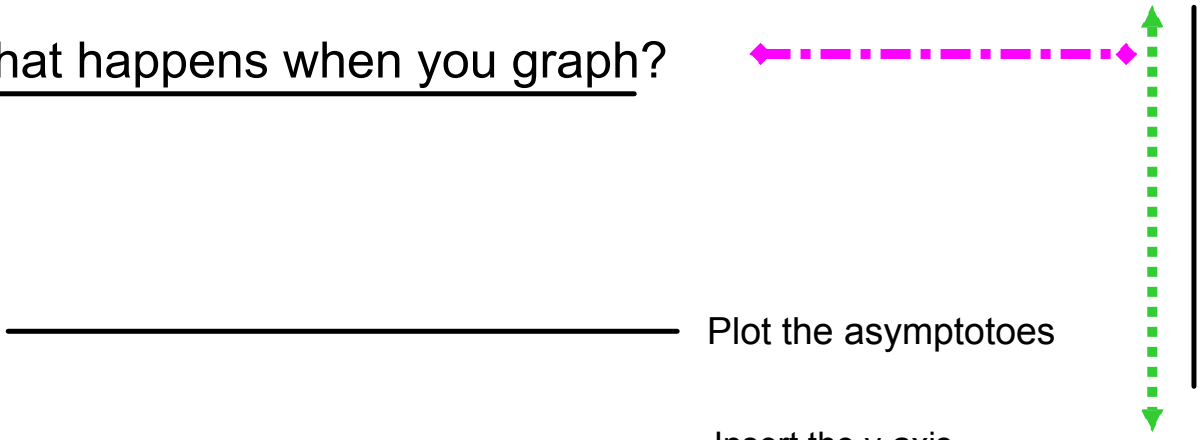
$\dfrac{3n+2}{7n+2}$; $\left\{0, 1, -\dfrac{2}{7}\right\}$

8) $\dfrac{2m^2-12m-80}{5m+20}$

$\dfrac{2(m-10)}{5}$; $\{-4\}$

Feb 11-2:55 PM

What happens when you graph?



Plot the asymptotoes

Insert the y-axis

Sketch the graph of the function

Feb 11-2:58 PM

Graphing a Rational Function

Vertical Asympototes

Horizontal Asympototes

Feb 11-8:23 PM

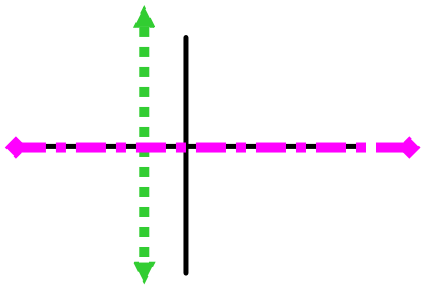
Graphing Rational Functions

What is the end behavior?

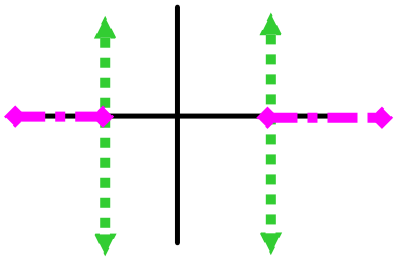
What is the highest degree of the rational function?

How many vertical asymptotes are there?

1 vertical asymptotes: _____

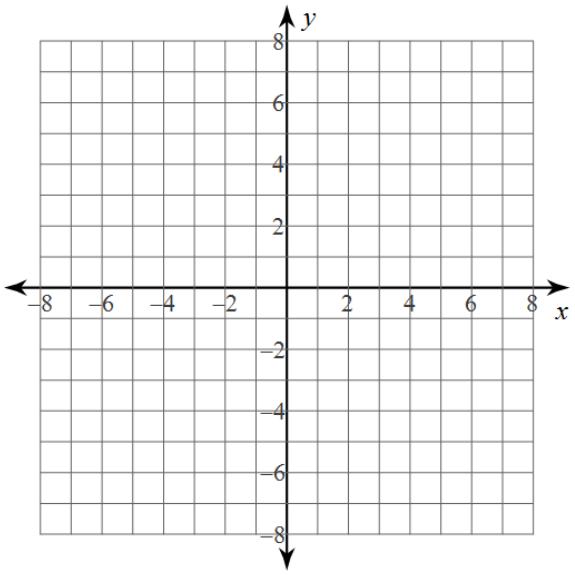


2 vertical asymptotes: _____



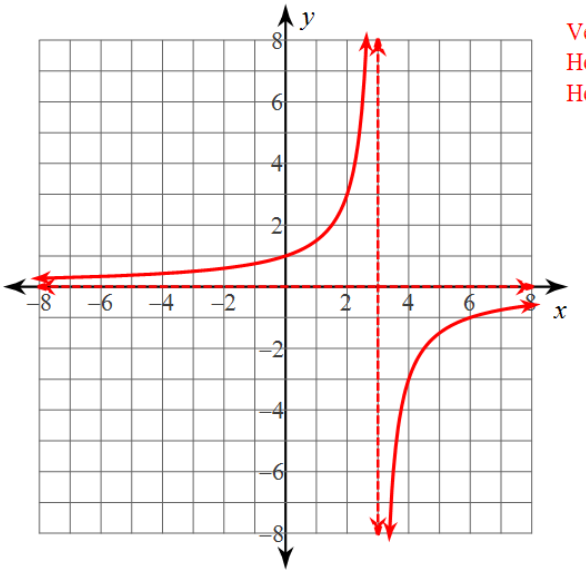
Feb 11-8:24 PM

1) $f(x) = -\dfrac{3}{x-3}$



Feb 11-8:28 PM

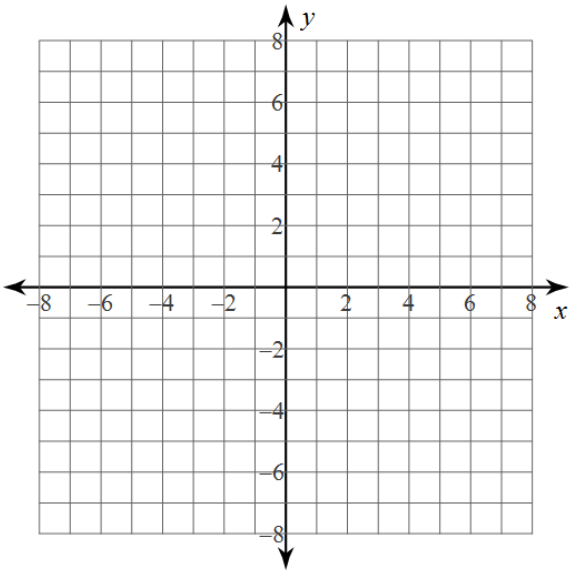
1) $f(x) = -\dfrac{3}{x-3}$



Vertical Asym.: $x = 3$
Holes: None
Horz. Asym.: $y = 0$

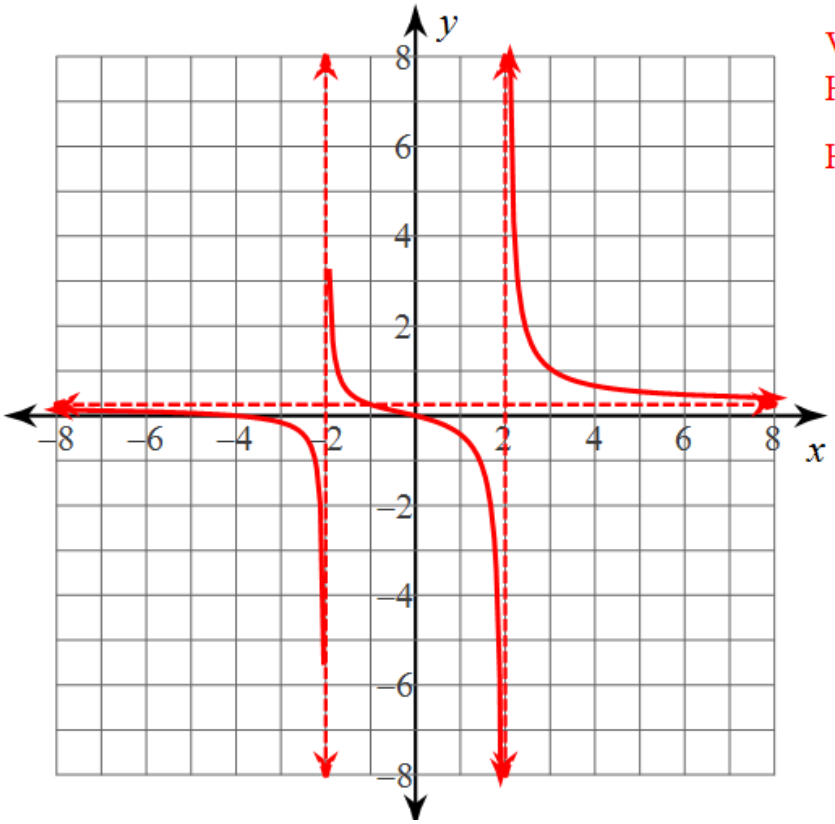
Feb 11-8:31 PM

2) $f(x) = \dfrac{x^2 + 4x}{4x^2 - 16}$



Feb 11-8:31 PM

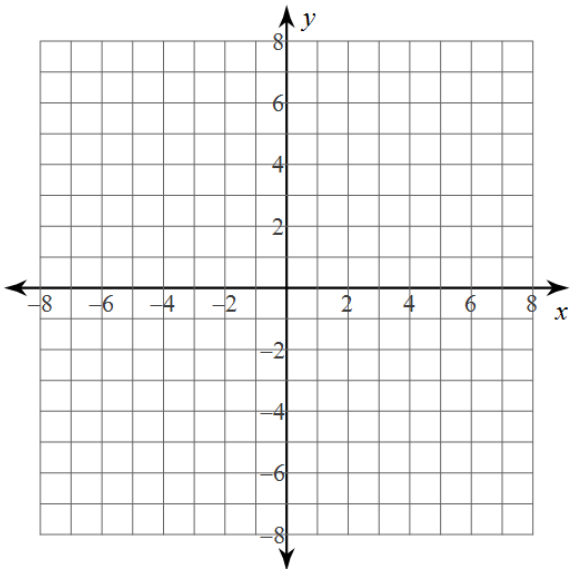
2) $f(x) = \dfrac{x^2 + 4x}{4x^2 - 16}$



Vertical Asym.: $x = 2$, $x = -2$
Holes: None
Horz. Asym.: $y = \dfrac{1}{4}$

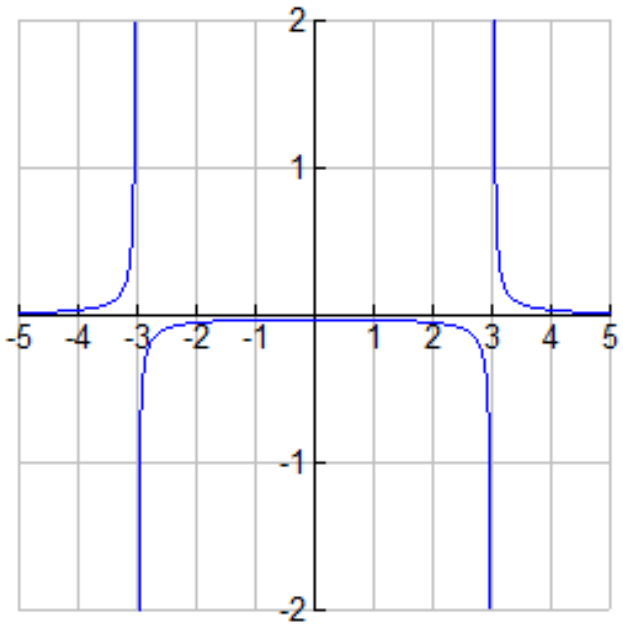
Feb 11-8:32 PM

3) $f(x) = \dfrac{1}{4x^2 - 36}$



Feb 11-8:32 PM

3) $f(x) = \dfrac{1}{4x^2 - 36}$

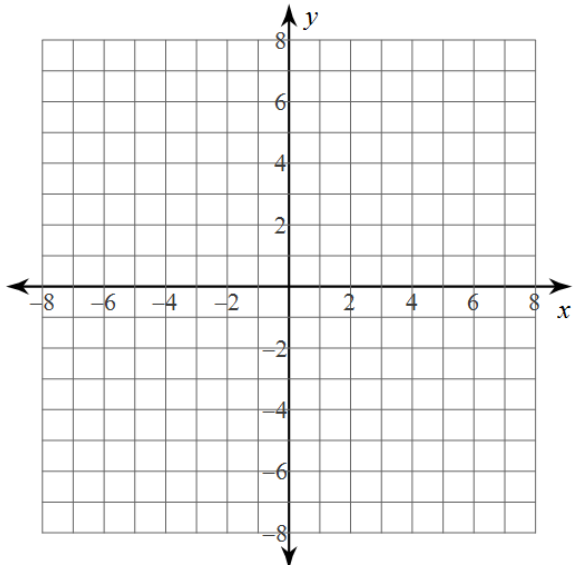


-2.

Vertical Asym.: $x = 3$, $x = -3$
Holes: None
Horz. Asym.: $y = 0$

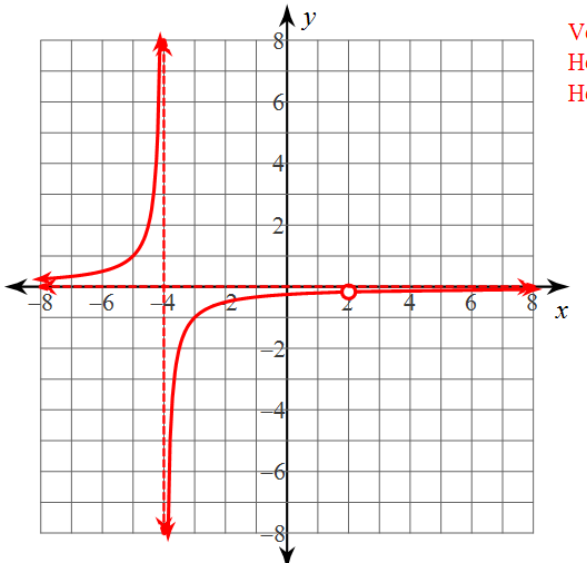
Feb 11-8:32 PM

4) $f(x) = \dfrac{x-2}{-x^2-2x+8}$



Feb 11-8:35 PM

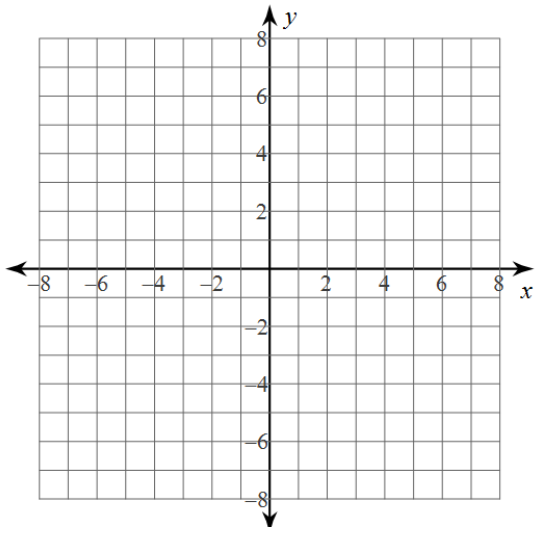
4) $f(x) = \dfrac{x-2}{-x^2-2x+8}$



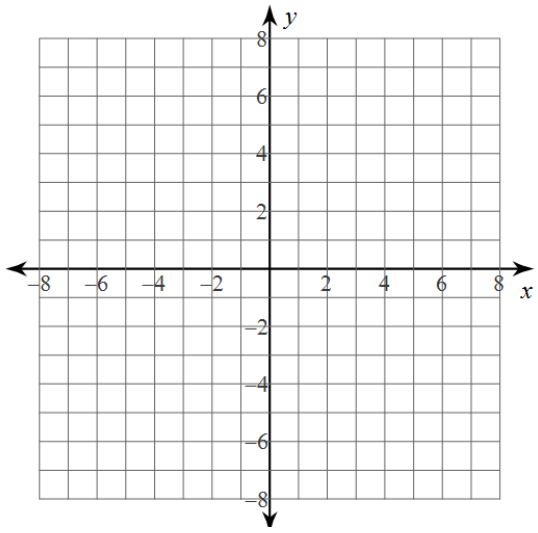
Vertical Asym.: $x = -4$
Holes: $x = 2$
Horz. Asym.: $y = 0$

Feb 11-8:37 PM

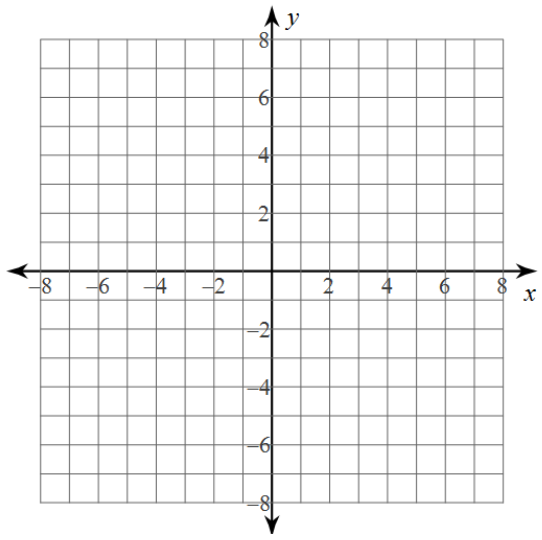
1) $f(x) = -\dfrac{x}{x^2 - 4x}$



2) $f(x) = -\dfrac{3}{x^2 - 2x - 3}$



3) $f(x) = \dfrac{x^2 + 4x}{x^2 - 2x - 3}$



4) $f(x) = \dfrac{-2x + 4}{x + 2}$