

Rational Inequalities

Find the critical values, make and shade a number line, and write the interval.

$$\frac{(x-2)(x+1)}{x-5} \leq 0$$

Interval Answer $(-\infty, -1] \cup [2, 5)$

$$\frac{x^2+1}{x^2-x-2} > 0$$

Interval Answer $(-\infty, -1] \cup [2, \infty)$

$$\frac{x+6}{x+1} > 2$$

Interval Answer $(-1, 4]$

$$\frac{2}{x+3} \leq \frac{1}{x-1}$$

Interval Answer $(-\infty, -3) \cup (1, 5]$

Solving Rational Functions:

Find the value for the variable in each function below.

$$\frac{2b-3}{7} = \frac{b+3}{2} = \frac{b+3}{14}$$

Answers: $-\frac{9}{4} = b$

$$\frac{5k}{k+2} + \frac{2}{k} = 5$$

$\frac{1}{2} = k$

$$\frac{4}{k^2-8k+12} = \frac{k}{k-2} + \frac{1}{k-6}$$

$k = 6$ or -1
Since "k" cannot equal "6" the solution is "-1"

$$x - \frac{2}{x-3} = \frac{x-1}{3-x}$$

$x = 3$ or $x = -1$
Since "x" cannot equal 3, the only solution is $x = -1$

Simplify Each Expression

$$\frac{5}{3x} - \frac{3}{2x^2 - 24x + 72}$$

Answers: $\frac{10x^2 - 129x + 360}{6x(x-6)^2}$

$$\frac{2x}{3} + \frac{4x}{2x^2 - 14x^2 + 12x}$$

$$\frac{2x^3 - 14x^2 + 12x + 6}{3(x-6)(x-1)}$$

$$\frac{5}{3x+15} + \frac{6x}{x+6}$$

$$\frac{95x+30+18x^2}{3(x+6)(x+5)}$$

$$\frac{a+2}{4a^2-18a^2+8a} - \frac{4a}{3a}$$

$$\frac{-29a+6-16a^3+72a^2}{6a(a-4)(2a-1)}$$

Rational Function Inverse

$$f(x) = \frac{x+5}{x-5}$$

Answers: $f^{-1}(x) = \frac{5x+5}{x-1}$

$$f(x) = \frac{-2x+7}{x}$$

$$f^{-1}(x) = \frac{7}{x+2}$$

$$f(x) = \frac{-3x-1}{x+5}$$

$$f^{-1}(x) = \frac{-5x-1}{x+3}$$

$$f(x) = \frac{x+7}{x}$$

$$f^{-1}(x) = \frac{7}{x-1}$$

$$\frac{x^2+1}{x^2-x-2} > 0$$

$$x^2+1 = 0 \Rightarrow x = \pm\sqrt{-1}$$

imaginary

$$x^2-1 = 0 \Rightarrow (x-1)(x+1)$$

$$x^2-1$$

Bottom: $x^2-x-2=0$
 $(x-2)(x+1)=0$
 $x=2, x=-1$
 $(-\infty, -1) \cup (2, \infty)$
 TP: $-2, -1, 2, 3$

$$\frac{x^2+1}{(x-2)(x+1)} > 0$$

$$\frac{4+1}{(-4)(-1)} = \frac{5}{4} > 0$$

$$\frac{0+1}{(-2)(1)} = \frac{1}{-2} > 0$$

$$\frac{9+1}{(1)(4)} = \frac{10}{4} > 0$$

$$\frac{2}{x+3} \leq \frac{1}{x-1}$$

$$\frac{2}{x+3} - \frac{1}{x-1} \leq 0$$

$$\frac{2(x-1) - 1(x+3)}{(x+3)(x-1)} \leq 0$$

$$\frac{2x-2-x-3}{(x+3)(x-1)} \leq 0$$

$$\frac{(x-5)}{(x+3)(x-1)} \leq 0$$

 $x-5=0 \Rightarrow x=5$

 $x+3=0 \Rightarrow x=-3$

 $x-1=0 \Rightarrow x=1$

$$\frac{x-5}{(x+3)(x-1)} \leq 0$$

$$\frac{-9}{(-4)(-5)} = \frac{-9}{20} < 0$$

$$\frac{-5}{(3)(-1)} = \frac{-5}{-3} = \frac{5}{3} < 0$$

$$\frac{-9}{5} < 0$$

$$\frac{-5}{(3)(1)} = \frac{-5}{3} < 0$$

$$\frac{4}{k^2 - 8k + 12} = \frac{k}{k-2} + \frac{1}{k-6}$$

$(k-2)(k-6)$

$$\frac{4}{(k-2)(k-6)} = \frac{k(k-6) + (k-2)}{(k-2)(k-6)}$$

$k \neq 6$
 $k \neq 2$

$$\frac{4}{(k-2)(k-6)} = \frac{k^2 - 5k - 2}{(k-2)(k-6)}$$

$$4 \cdot \frac{(k-2)(k-6)}{(k-2)(k-6)} = \frac{(k^2 - 5k - 2)(k-2)(k-6)}{(k-2)(k-6)}$$

$$4 = k^2 - 5k - 2$$

↙

$$0 = k^2 - 5k - 6$$

$$0 = (k-6)(k+1)$$

$$k = 6 \quad k = -1$$

$$\frac{5}{3x+15} + \frac{6x}{x+6}$$

$[5(x+6) + 6x(3x+15)]$
 $(3x+15)(x+6)$

$$\frac{5x+30 + 18x^2 + 90x}{(3x+15)(x+6)}$$

$$\frac{18x^2 + 95x + 30}{(3x+15)(x+6)}$$

$$\frac{5 \cdot 2 \cdot (x-6)(x-6) - 3x \cdot 3}{(3x)(2)(x-6)(x-6)}$$

$10(x^2 - 12x + 36) - 9x$

$$\frac{10x^2 - 120x + 360 - 9x}{(3x)(2)(x-6)(x-6)}$$

$10 \quad -126 \quad 360$
 $x=6$

$$\frac{10x^2 - 129x + 360}{6x(x-6)(x-6)}$$

$$f(x) = \frac{-2x+7}{x}$$

$$y = \frac{-2x+7}{x}$$

$$\frac{x}{1} = \frac{-2y+7}{y}$$

$$xy = -2y + 7$$

$$\begin{array}{r} +2y \\ \hline xy + 2y = 7 \end{array}$$

$$y(x+2) = 7$$

$$\frac{y(x+2)}{x+2} = \frac{7}{x+2}$$

$$y = \frac{7}{x+2}$$

$$x = \frac{-3y - 1}{y + 5}$$

$$xy + 5x = -3y - 1$$

$$xy + 3y = -5x - 1$$

$$y(x+3) = -5x - 1$$

$$y = \frac{-5x - 1}{x + 3} \text{ or } \frac{5x + 1}{-x - 3}$$

$$x - \frac{2}{x-3} = \frac{x-1}{3-x}$$

$$x \frac{(x-3)(x-1)}{(x-3)(3-x)} + \frac{2}{x-3} \frac{(3-x)(3-x)}{(3-x)}$$

$$x = \frac{(x^2 - 4x + 3) + 2(3-x)}{(x-3)(3-x)}$$

$$\frac{x}{1} = \frac{x^2 - 6x + 9}{(x-3)(3-x)}$$

$$\frac{x}{1} = \frac{(x-3)}{(3-x)}$$

$$x(3-x) = x-3$$

$$-3x - x^2 = x - 3$$

$$0 = x^2 - 2x - 3$$

$$0 = (x-3)(x+1)$$

$x \neq 3$ $x = -1$

$$\frac{4}{k^2 + 8k + 12} = \frac{k}{k-2} + \frac{1}{k-6}$$

$$\frac{4}{(k-2)(k-6)} = \frac{k(k-6) + 1(k-2)}{(k-2)(k-6)}$$

$$\frac{4}{(k-2)(k-6)} = \frac{k^2 - 6k + k - 2}{(k-2)(k-6)} \quad \begin{matrix} k \neq 6 \\ k \neq 2 \\ k-2=0 \\ k-6=0 \end{matrix}$$

$$4(k-2)(k-6) = (k^2 - 5k - 2)(k-2)(k-6)$$

$$4 = k^2 - 5k - 2$$

$$0 = k^2 - 5k - 6$$

$$0 = (k-6)(k+1)$$

$$k-6=0 \quad k+1=0$$

$k \neq 6$ $k = -1$

$$\frac{2}{x+3} \leq \frac{1}{x-1}$$

$$\frac{2(x-1)}{(x+3)(x-1)} - \frac{1(x+3)}{(x+3)(x-1)} \leq 0$$

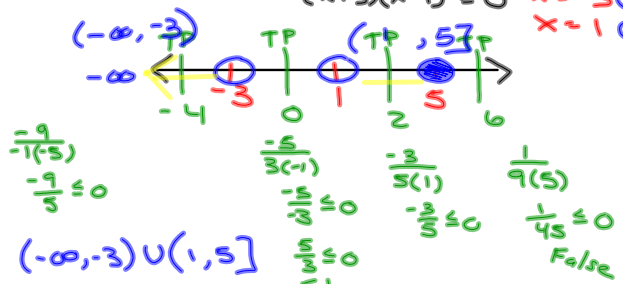
$$\frac{2(x-1) - 1(x+3)}{(x+3)(x-1)} \leq 0$$

$$\frac{2x - 2 - x - 3}{(x+3)(x-1)} \leq 0$$

$$\frac{x-5}{(x+3)(x-1)} \leq 0$$

$$x-5=0 \quad x=5$$

$$(x+3)(x-1)=0 \quad x=-3 \quad x=1$$



$$y = \frac{-2x+7}{x}$$

$$x = \frac{-2y+7}{y}$$

$$xy = -2y + 7$$

$$\begin{array}{r} +2y \quad \leftarrow \\ \hline \end{array}$$

$$xy + 2y = 7$$

$$y(x+2) = 7$$

$$\frac{y(x+2)}{(x+2)} = \frac{7}{(x+2)}$$

$$y = \frac{7}{x+2}$$

$$\frac{5}{3x} - \frac{3}{2x^2 - 24x + 72} \longrightarrow \frac{10x^2 - 129x + 360}{6x(x-6)^2}$$

$$\frac{2x}{3} + \frac{4x}{2x^2 - 14x^2 + 12x} \longrightarrow \frac{2x^3 - 14x^2 + 12x + 6}{3(x-6)(x-1)}$$

$$\frac{5}{3x+15} + \frac{6x}{x+6} \longrightarrow \frac{95x + 30 + 18x^2}{3(x+6)(x+5)}$$

$$\frac{a+2}{4a^3 - 18a^2 + 8a} - \frac{4a}{3a} \longrightarrow \frac{-29a + 6 - 16a^3 + 72a^2}{6a(a-4)(2a-1)}$$

$$\frac{5}{3x} - \frac{3}{2x^2 - 24x + 72}$$

$$\frac{5}{3x} - \frac{3}{2(x^2 - 12x + 36)}$$

$$\frac{5}{3x} - \frac{3}{2(x-6)(x-6)}$$

$$\frac{5}{3x} - \frac{3}{2(x-6)(x-6)}$$

$$\frac{10(x^2 - 12x + 36) - 9x}{3x(2)(x-6)(x-6)}$$

$$\frac{10x^2 - 120x + 360 - 9x}{6x(x-6)(x-6)}$$

$$\frac{10x^2 - 129x + 360}{6x(x-6)(x-6)}$$

$$\begin{array}{r} x-6=0 \\ x=6 \quad | \quad 10 \quad -129 \quad 360 \\ \downarrow \quad 60 \quad -414 \\ \hline 10 \quad -69 \end{array}$$

$$y = \frac{-3x-1}{x+5}$$

$$x = \frac{-3y-1}{y+5}$$

$$xy + 5x = -3y - 1$$

$$xy + 3y = -5x - 1$$

$$y(x+3) = -5x - 1$$

$$y = \frac{-5x-1}{x+3} \quad \text{or} \quad \frac{5x+1}{-x-3}$$

$$\frac{x+6}{x+1} > 2$$

$$\frac{x+6}{x+1} - \frac{2}{1} > 0$$

$$\frac{x+6-2(x+1)}{x+1} > 0$$

$$\frac{x+6-2x-2}{x+1} > 0$$

$$\frac{-x+4}{x+1} > 0$$

Top: $-x+4=0$
 $x=4$
 Bottom: $x+1=0$
 $x=-1$
 open circle

$5/0 > 0$ No
 $0/6 > 0$ open circle
 $2/3 > 0$
 $3/8 > 0$
 $-6/7 > 0$
 $-2/7 > 0$ No
 False

$$\frac{5k}{(k+2)(k)} + \frac{2}{(k)(k+2)} = \frac{5}{1}$$

$$\frac{5k^2+2k+4}{(k+2)(k)} = \frac{5}{1}$$

$$5k^2+2k+4 = 5(k+2)(k)$$

$$5k^2+2k+4 = 5(k^2+2k)$$

$$5k^2+2k+4 = 5k^2+10k$$

$$2k+4 = 10k$$

$$4 = 8k$$

$$\frac{1}{2} = \frac{4}{8} = k$$

$\frac{5k}{k+2} = \frac{5}{1} - \frac{2}{k}$

$$y = \frac{x+5}{x-5}$$

$$x = \frac{y+5}{y-5}$$

$$x(y-5) = y+5$$

$$xy - 5x = y+5$$

$$xy - y = 5x+5$$

$$y(x-1) = 5x+5$$

$$\frac{y(x-1)}{(x-1)} = \frac{5x+5}{x-1}$$

$$y = \frac{5x+5}{x-1} = \frac{5(x+1)}{x-1}$$

$$y = \frac{-3x-1}{x+5}$$

$$x = \frac{-3y-1}{y+5}$$

$$x(y+5) = -3y-1$$

$$xy + 5x = -3y-1$$

$$xy + 3y = -5x-1$$

$$y(x+3) = \frac{-5x-1}{x+3}$$

$$y = \frac{-5x-1}{x+3} \text{ or } \frac{5x+1}{-x-3}$$

$$\frac{5(x+6)}{3x+15} + \frac{6x(3x+15)}{x+6(3x+15)}$$

$$\frac{5(x+6) + 6x(3x+15)}{(3x+15)(x+6)}$$

$$\frac{5x+30 + 18x^2 + 90x}{(3x+15)(x+6)}$$

$$\frac{18x^2 + 95x + 30}{3(x+5)(x+6)}$$

$$\frac{x^2+1}{x^2-x-2} > 0$$

$$x^2+1=0$$

$$x^2 = -1$$

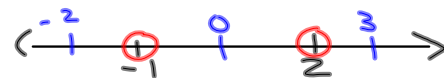
$$x = \pm\sqrt{-1}$$

imaginary

$$x^2-x-2=0$$

$$(x-2)(x+1)=0$$

$$x=2 \quad x=-1$$



$$x - \frac{2}{x-3} = \frac{x-1}{3-x}$$

$$\frac{x}{1} = \frac{x-1(x-2)}{(3-x)(x-3)} + \frac{2(3-x)}{(3-x)(3-x)}$$

$$\frac{x}{1} = \frac{(x-1)(x-3) + 2(3-x)}{(3-x)(x-3)}$$

$$\frac{x}{1} = \frac{x^2 - 4x + 3 + 6 - 2x}{(3-x)(x-3)}$$

$$\frac{x}{1} = \frac{x^2 - 6x + 9}{(3-x)(x-3)}$$

$$\frac{x}{1} = \frac{(x-3)}{(3-x)}$$

$$x-3 = x(3-x)$$

$$x-3 = 3x-x^2$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x+3 \quad x=-1$$

$$\frac{5}{3x} - \frac{3}{2x^2 - 21x + 72}$$

Get out your reviews.....?

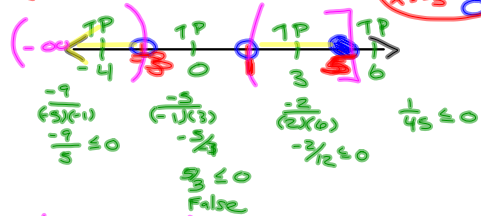
$$\frac{2}{x+3} \leq \frac{1}{x-1}$$

$$\frac{2}{x+3} - \frac{1}{x-1} \leq 0$$

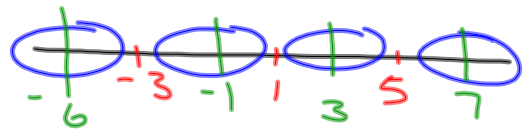
$$\frac{2x-2-x-3}{(x-1)(x+3)} \leq 0$$

$$\frac{x-5}{(x-1)(x+3)} \leq 0$$

$x-5=0 \Rightarrow x=5$ ●
 $(x-1)(x+3)=0 \Rightarrow x=1$ ○
 $x=-3$ ○



$$(-\infty, -3) \cup (1, 5]$$



$$\frac{4}{k^2-8k+12} = \frac{k}{k-2} + \frac{1}{k-6}$$

$$\frac{4}{(k-2)(k-6)} = \frac{k(k-6) + 1(k-2)}{(k-2)(k-6)}$$

$$\frac{4}{(k-2)(k-6)} = \frac{k^2-6k+k-2}{(k-2)(k-6)}$$

$$\frac{4}{(k-2)(k-6)} = \frac{k^2-5k-2}{(k-2)(k-6)}$$

$$\frac{4(k-2)(k-6)}{(k-2)(k-6)} = \frac{(k-2)(k-6)(k^2-5k-2)}{(k-2)(k-6)}$$

$$4 = k^2-5k-2$$

$$0 = k^2-5k-6$$

$$0 = (k-6)(k+1)$$

$$(k \neq 6) \quad k = -1$$

answer k = -

k ≠ 6
k ≠ 2

1. FACTOR

$$\frac{5}{3x} - \frac{3}{2x^2 - 24x + 72}$$

$$\frac{5}{3x} - \frac{3}{2(x^2 - 12x + 36)}$$

$$\frac{5}{3x} - \frac{3}{2(x-6)(x-6)}$$

$$\frac{5(2)(x-6)(x-6) - 3(3x)}{(3x)(2)(x-6)(x-6)}$$

$$5(2)(x-6)(x-6) - 9x$$

$$10(x-6)(x-6) - 9x$$

$$10(x^2 - 12x + 36) - 9x$$

$$10x^2 - 120x + 360 - 9x$$

$$\frac{10x^2 - 129x + 360}{(3x)(2)(x-6)(x-6)}$$

$$\frac{10x^2 - 129x + 360}{6x(x-6)^2}$$

$$\frac{10x^2 - 129x + 360}{6x(x-6)^2}$$

$$\begin{array}{r} x-6 \\ x=6 \end{array} \begin{array}{r} 10 \quad -129 \quad 360 \\ \underline{60} \quad 414 \\ 10 \quad 69 \end{array}$$

$$\frac{5}{3x+15} + \frac{6x}{x+6}$$

$$\frac{5(x+6) + 6x(3x+15)}{(3x+15)(x+6)}$$

$$\frac{5x+30 + 18x^2 + 90x}{(3x+15)(x+6)}$$

$$\frac{18x^2 + 95x + 30}{(3x+15)(x+6)}$$

$$\frac{18x^2 + 95x + 30}{3(x+5)(x+6)}$$

$b = \frac{1}{4}$ ①
 $k = \frac{1}{2}$ ②
 ~~$k = -1$~~ or $k = -1$ ✓
 $x = -1$ ③

$$\frac{2b-3}{7} \rightarrow \frac{b}{2} = \frac{b+3}{14}$$

$$\frac{2(2b-3) - b(7)}{14} = \frac{b+3}{14}$$

$$\frac{4b-6-7b}{14} = \frac{b+3}{14}$$

$$\frac{-3b-6}{14} = \frac{b+3}{14}$$

$$\frac{14(-3b-6)}{14} = \frac{14(b+3)}{14}$$

$$-3b-6 = b+3$$

$$\begin{array}{r} +3b \quad +3b \\ -6 = 4b+3 \\ -3 \quad \leftarrow -3 \\ \hline -9 = 4b \\ -9/4 = b \end{array}$$

$$\frac{4}{k^2-8k+12} = \frac{(k-2)k}{(k-2)(k-6)} + \frac{1(k-2)}{(k-6)(k-2)}$$

$$\frac{4}{(k-2)(k-6)} = \frac{k^2-6k+k-2}{(k-2)(k-6)}$$

$$\frac{4}{(k-2)(k-6)} = \frac{k^2-5k-2}{(k-2)(k-6)}$$

$$\frac{4(k-2)(k-6)}{(k-2)(k-6)} = \frac{(k-2)(k-6)(k^2-5k-2)}{(k-2)(k-6)}$$

$$4 = k^2 - 5k - 2$$

$$0 = k^2 - 5k - 6$$

$$0 = (k-6)(k+1)$$

$k=6$ $k=-1$

$k = -1$

$$\frac{a+2}{4a^3-18a^2+8a} - \frac{4a}{3a}$$

$$\frac{a+2}{4a^3-18a^2+8a} - \frac{4a}{3a}$$

$$\frac{3(a+2)}{3 \cdot 2a(2a^2-9a+4)} - \frac{4(2a)(2a^2-9a+4)}{3(2a)(2a^2-9a+4)}$$

$$\frac{3a+6 - 8a(2a^2-9a+4)}{6a(2a^2-9a+4)}$$

$$\frac{3a+6 - 16a^3 + 72a^2 - 32a}{6a(2a^2-9a+4)}$$

$$\frac{-16a^3 + 72a^2 - 33a + 6}{6a(2a^2-9a+4)}$$

answer

$$\frac{2}{x+3} \leq \frac{1}{x-1}$$

$$\frac{2(x-1)}{(x+3)(x-1)(x+3)} \leq \frac{1(x+3)}{(x-1)(x+3)} \leq 0$$

$$\frac{2(x-1) - 1(x+3)}{(x+3)(x-1)} \leq 0$$

$$\frac{2x - 2 - x - 3}{(x+3)(x-1)} \leq 0$$

$$\frac{(x-5)}{(x+3)(x-1)} \leq 0$$

$$x-5=0 \quad x=5$$

$$(x+3)(x-1)=0 \quad x=-3$$

$$x=1$$

