

Identify the points of discontinuity, holes, vertical asymptotes, x-intercepts, and horizontal asymptote of each.

1) $f(x) = \frac{1}{3x^2 + 3x - 18}$
 $3(x^2 + x - 6)$

2) $f(x) = \frac{x-2}{x-4}$ *No holes*
 VA: $x=4$
 HA: $y=1$

3) $f(x) = \frac{x^3 - x^2 - 6x}{-3x^2 - 3x + 18}$ HA: top slant

4) $f(x) = \frac{x^2 + x - 6}{-4x^2 - 16x - 12}$

Holes: None
 Restrictions

VA:

$-3(x+3)(x-2) = 0$

$\frac{-3(x+3)(x-2)}{-3} = \frac{0}{-3}$

$(x+3)(x-2) = 0$

$x+3=0 \quad x-2=0$

$x=-3 \quad x=2$

$\frac{x(x^2 - x - 6)}{-3(x^2 + x - 6)} \quad \frac{x(x-3)(x+2)}{-3(x+3)(x-2)}$

4) $f(x) = \frac{x^2 + x - 6}{-4x^2 - 16x - 12}$

HA: look at degree
 SAME $\frac{1x^2 \dots}{-4x^2 \dots} \Rightarrow y = -\frac{1}{4}$

$-1 \cdot 6$ $1 \cdot -6$
 $-2 \cdot 3$ $2 \cdot -3$

Restrictions

Holes: ~~cancel~~
 $\frac{(x-2)}{-4(x+1)} = \frac{-5}{8}$ $x+3=0$
 $x=-3$
 $(-3, -\frac{5}{8})$

$\frac{(x+3)(x-2)}{-4(x^2+4x+3)} = \frac{\cancel{(x+3)}(x-2)}{-4\cancel{(x+3)}(x+1)}$

\uparrow
 $1 \cdot 3$
 $-1 \cdot -3$

VA: $\frac{-4(x+1)}{-4} = 0$
 $x+1=0$
 $x=-1$

1) $f(x) = \frac{1}{3x^2 + 3x - 18}$ ↙ HA: bottom $y=0$

$3(x^2 + x - 6)$

$3(x+3)(x-2)$

FACTORS

-1 · 6	1 · -6
-2 · 3	2 · -3

$$f(x) = \frac{1}{3(x+3)(x-2)}$$

No holes → cancel out

$$\frac{\cancel{3}(x+3)(x-2)}{\cancel{3}} = \frac{0}{3}$$

VA: $3(x+3)(x-2) = 0$

$x+3=0$ $x-2=0$

$x=-3$ $x=2$

} Restrictions

5) $f(x) = -\frac{4}{x^2 - 3x}$

HA: bottom $y=0$

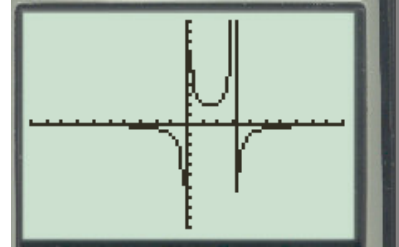
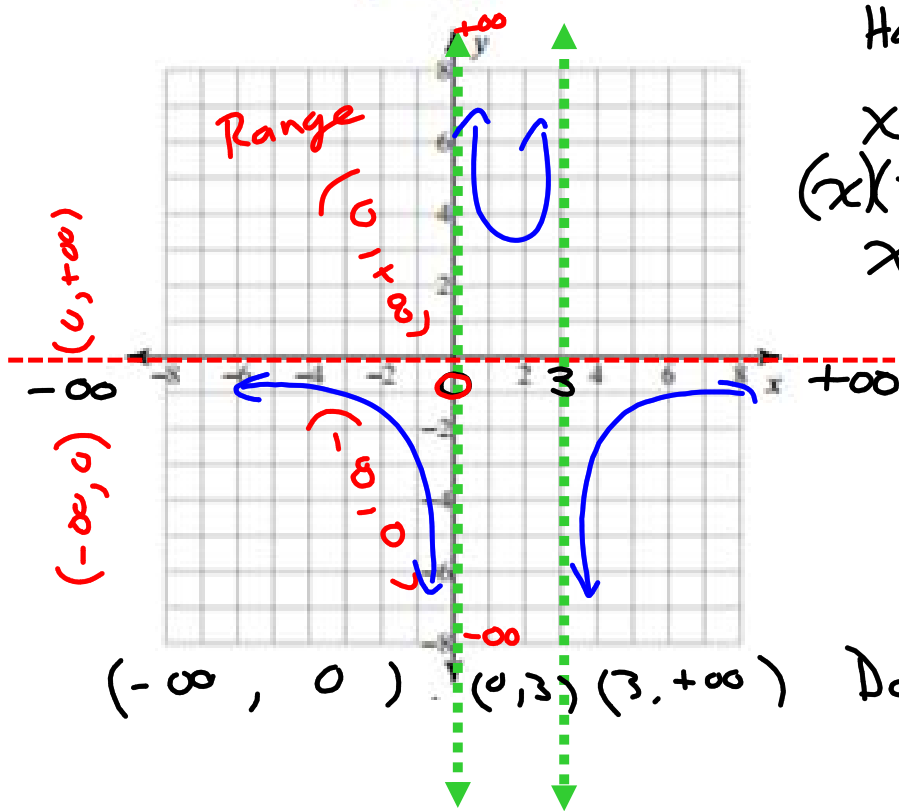
Holes & VA:

$$x^2 - 3x = 0$$

$$(x)(x - 3) = 0$$

$$x = 0 \quad x - 3 = 0$$

$$x = 3$$



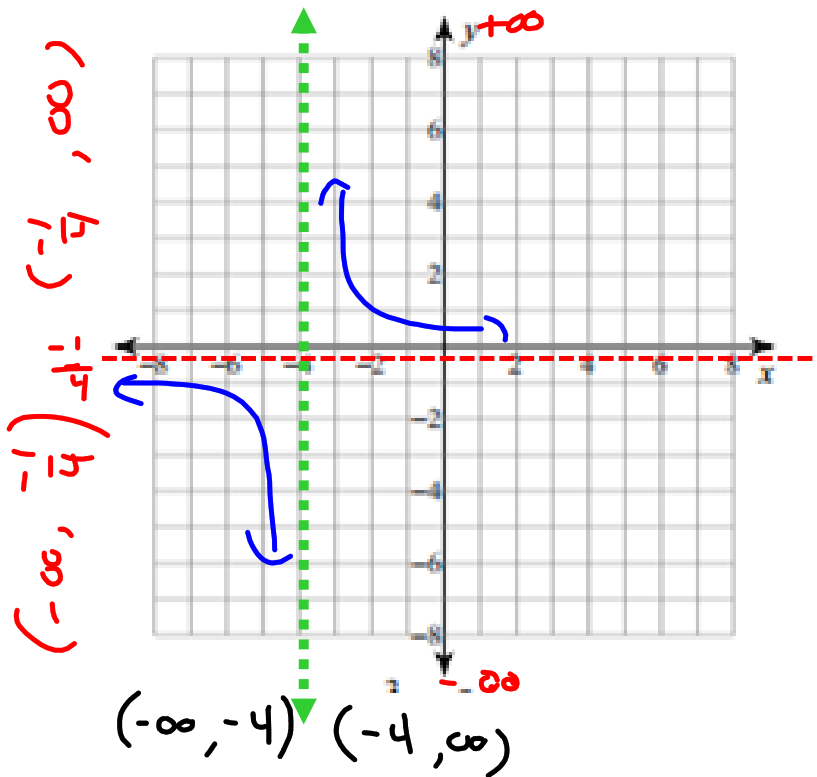
$(-\infty, 0) \cup (0, 3) \cup (3, +\infty)$ Domain

6) $f(x) = \frac{x-4}{-4x-16}$

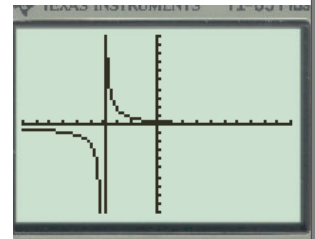
HA: $y = -\frac{1}{4}$

Holes ~~None~~
 $\frac{(x-4)}{-4(x+4)}$

VA:
 $-4(x+4) = 0$
 $(x+4) = 0$
 $x = -4$



Range: $(-\infty, -\frac{1}{4}) \cup (-\frac{1}{4}, \infty)$

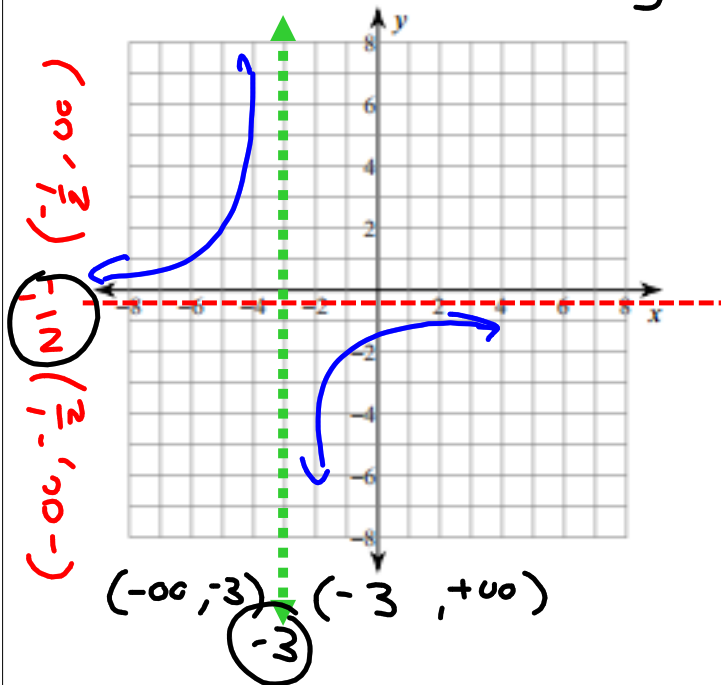


$$7) f(x) = \frac{x+4}{-2x-6}$$

HA: degrees are the same
 $y = -\frac{1}{2}$

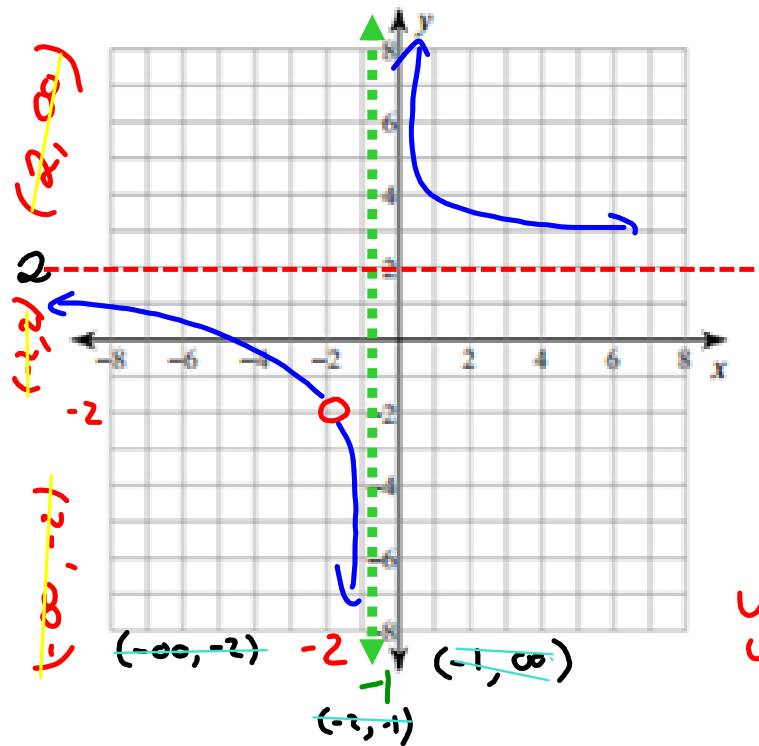
Holes: None
 $\frac{x+4}{-2(x+3)}$

VA: $-2(x+3) = 0$
 $x = -3$



8) $f(x) = \frac{2x^2 + 10x + 12}{x^2 + 3x + 2}$

HA: $y = 2$



Holes: $\frac{2(x^2 + 5x + 6)}{(x^2 + 3x + 2)}$

$\frac{2(x+3)(x+2)}{(x+1)(x+2)}$

$\frac{2(x+3)(\cancel{x+2})}{(x+1)(\cancel{x+2})}$

Hole @ $x+2=0$
 $x = -2$

$y = \frac{2(-2+3)}{(-2+1)} = \frac{2(1)}{-1} = -2$

$(-2, -2)$

VA: $(x+1) = 0$
 $x = -1$