Representing Polynomials

Projector Resources

Projector Resources   Representing Polynomials
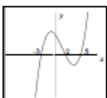
Day 1

Aug 27-10:02 PM

What do you know?

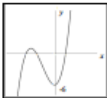### Cubic Graphs and Their Equations

1. Write down an equation of a cubic function that would give a graph like the one shown here. It crosses the x-axis at $(-3,0)$, $(2,0)$, and $(5,0)$.

2. Write down an equation of a cubic function that would give a graph like the one shown here. It crosses the y-axis at $(0,-6)$.

3. On the axes, sketch a graph of the function $y = (x+1)(x-4)^2$. You do not need to plot it accurately! Show where the graph crosses the x- and y-axes.

4. Write down the equation of the graph you get after you:
   (i)   Reflect $y = (x+1)(x-4)^2$ over the x-axis:
   (ii) Reflect $y = (x+1)(x-4)^2$ over the y-axis:
   (iii) Horizontally translate $y = (x+1)(x-4)^2$ through +2 units:
   (iv) Vertically translate $y = (x+1)(x-4)^2$ through +3 units:

Student Materials   Representing Polynomials   S-1
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Complete this worksheet
by yourself, about 20 min.

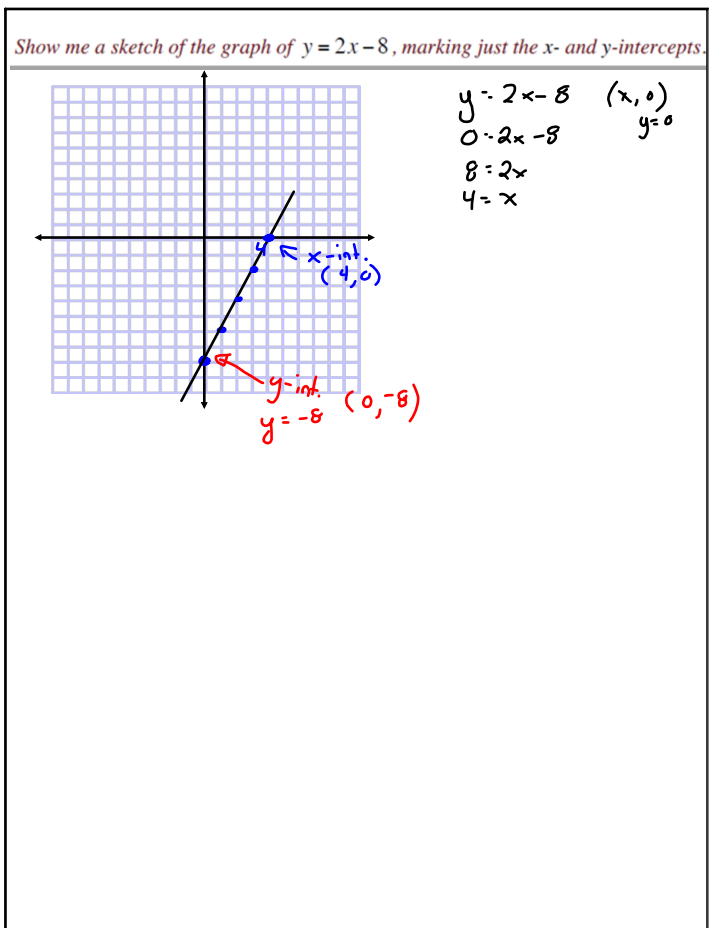Aug 27-2:18 PM

Teacher Lead Lesson

*could allow students to use white boards

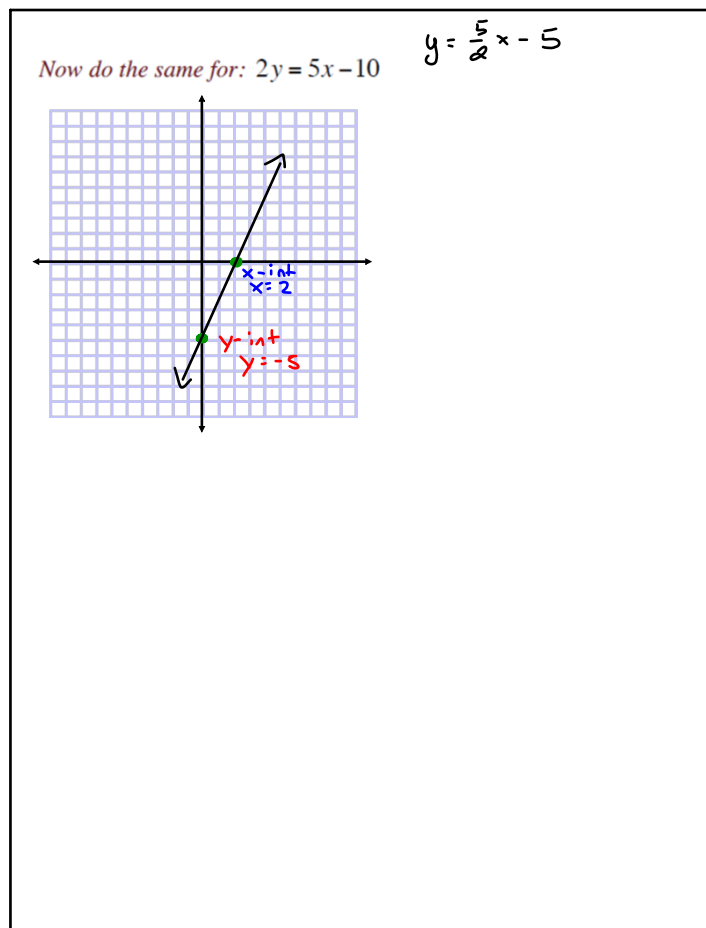Show me a sketch of the graph of $y = 2x - 8$, marking just the x- and y-intercepts.

Now do the same for: $2y = 5x - 10$

Show me a sketch of the graph of $y = (x-3)(x+4)$, marking the x- and y-intercepts.
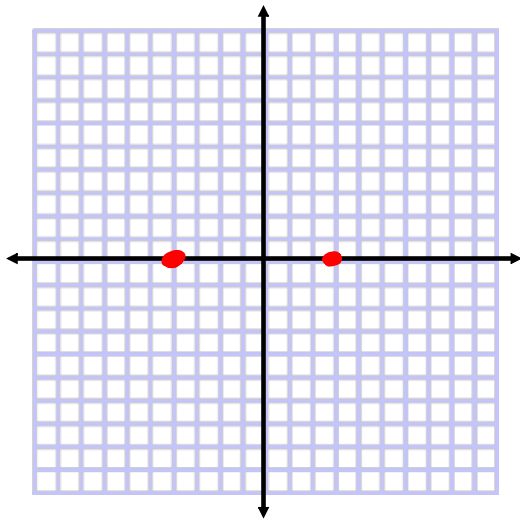
Aug 27-2:01 PM

Show me a sketch of the graph of $y = 2x - 8$, marking just the x- and y-intercepts.

$$y = 2x - 8 \quad (x, 0)$$
$$0 = 2x - 8 \quad y = 0$$
$$8 = 2x$$
$$4 = x$$

x-int.
(4, 0)

y-int.
y = -8   (0, -8)

Aug 28-8:42 AM

Now do the same for: $2y = 5x - 10$ $y = \frac{5}{2}x - 5$

x-int
x = 2

y-int
y = -5

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3

Show me a sketch of the graph of $y = (x-3)(x+4)$, *marking the x- and y-intercepts.*

$(x,0)$ x-int
$y = 0$

$0 = (x-3)(x+4)$

$(x-3)(x+4) = 0$
$x = 3$ $x = -4$

Aug 28-8:51 AM

Individually 10min. and then Small Group 10 min

$y = -(x-h)^2 + k$ ↻ $y = (x-h)^2 + k$ ↻

Show me an equation that fits:

$(x+1)(x-5)$
$y = (x+2)(x-1)$ $y =$ $-(x+8)(x-2)$ $y =$
                              $y =$

| Graph A | Graph B | Graph C | Graph D |

− +

-2 5
$x = -2$ $x = 5$
$(x+2)(x-5)$

1 5
$x = 1$ $x = 5$
$(x-1)(x-5)$

− +
-8 2
$(x+8)(x-2)$

-2
$(x+2)(x+2)$
$(x+2)^2$

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4
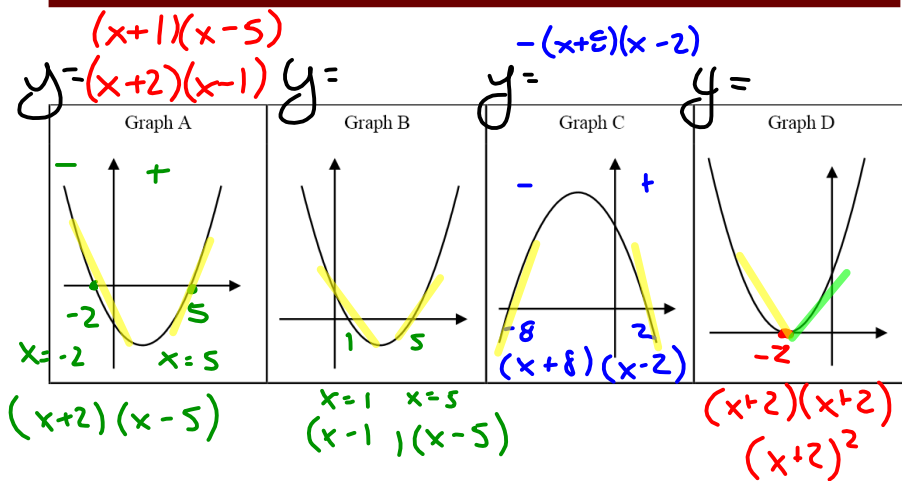
Day 2

Aug 27-10:02 PM

Match the Cards to the Graphs

Working Together

1. Take it in turns to match a function to its graph.

2. As you do this, label the graph to show the intercepts on the x- and y-axes.

3. If you match two cards, explain how you came to your decision.

4. If you don't agree or understand, ask your partner to explain their reasoning.

5. You **all** need to agree on, and explain the matching of every card.

6. You may find that there is more than one function that will fit some graphs!

7. If you have some functions left over, sketch graphs on the blank cards to match these functions.

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Aug 27-2:04 PM

Whole Class Discussion

Let's share how groups matched the graphs to equations...Please share your **reasoning** and then tell use your answer.
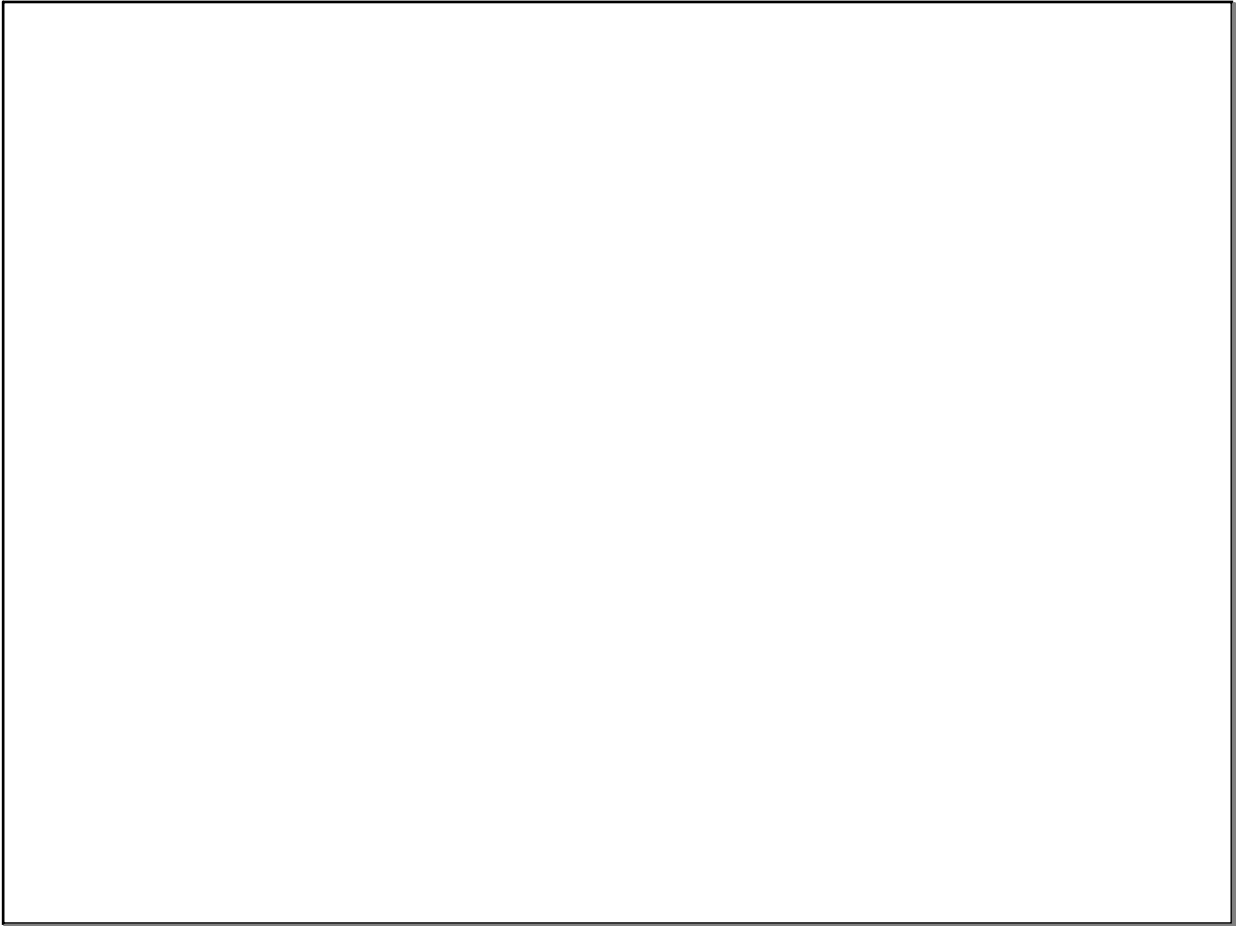
How did you decide to match this card?

Can someone else put that into their own words?

Aug 27-1:39 PM

What do these two functions have in common? What are their differences? How are these similarities and differences represented in the graph?
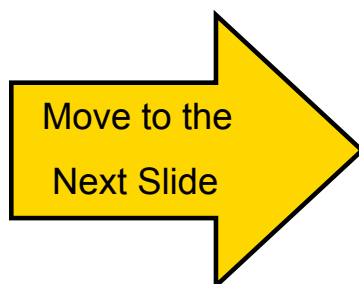
Common	Difference	Similarities

Aug 27-9:06 PM

Aug 29-8:44 AM

Matching Answers

Move to the

Next Slide

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Lesson tasks:

The correct matching of the graphs and functions is shown below:

Graph A	Function 2	Graph D	Function 6
	$y = (x+1)(x+2)(x+4)$		$y = x(x-3)^2$ **Function 9** $y = (x-1)^2(x-4)+4$
Graph B	**Function 1**	**Graph E**	**Function 3**
	$y = x(x-1)(x+2)$		$y = -(x+1)(x+2)(x+4)$
Graph C	**Function 4**	**Graph F**	**Function 8**
	$y = (x-1)^2(x-4)$		$y = -(x+1)^2(x+4)$

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The missing graphs are:

Function 5	
$y = -(x-1)^2(x-4)$	
Function 7 $y = -(x+1)^2(x+4)+4$ **Function 11** $y = -x(x+3)^2$	
Function 10 $y = (x-1)(x-2)(x-4)$	

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What do these two functions have in common? What are their differences? How are these similarities and differences represented in the graph?

Show me two graphs that are reflections of each other. Describe the reflection. How is this reflection represented in the function?

Show me two graphs that represent a translation of one graph onto another. Describe the translation. How is this translation represented in the function?
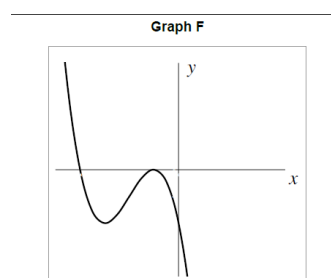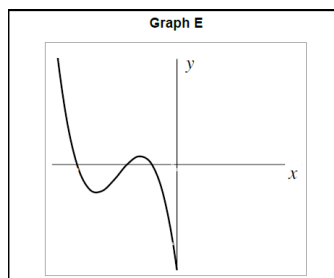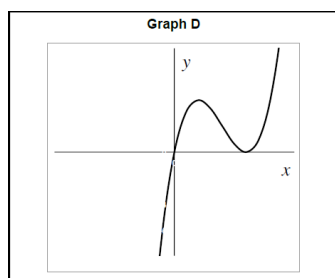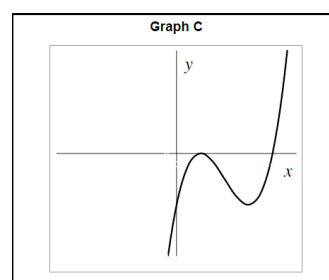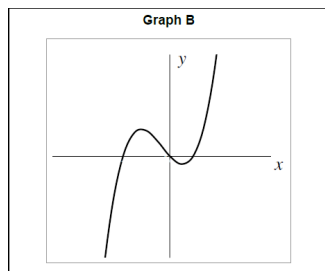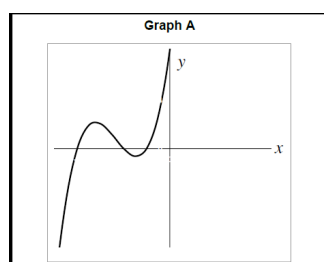
Aug 27-9:06 PM

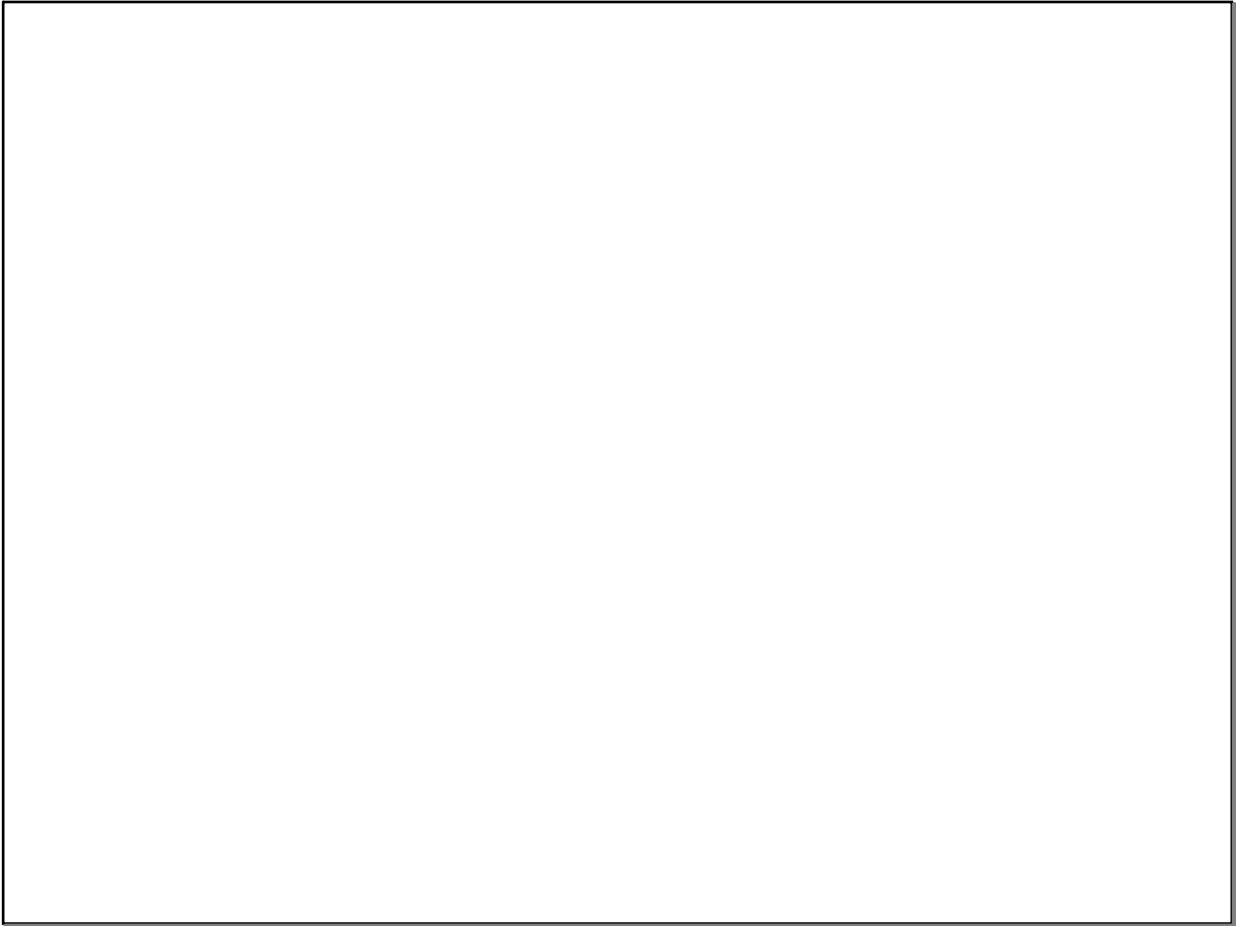*Show me two graphs that are reflections of each other. Describe the reflection. How is this reflection represented in the function?*

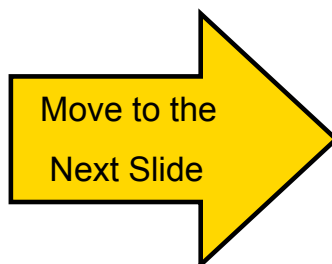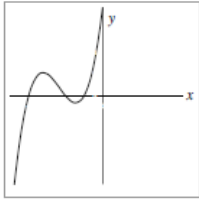*Drag each graph below to show pairs

Play Matching Game

Graph A

Graph B

Graph C

Graph D

Graph E

Graph F

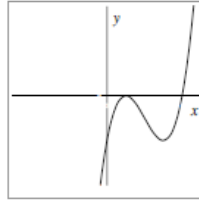Aug 27-9:08 PM
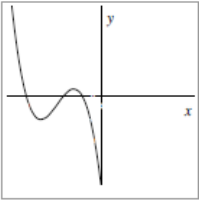
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Answer to Reflection Matching

Move to the
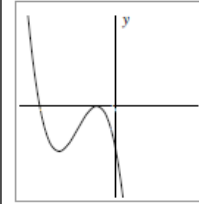
Next Slide

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Graph A	**Function 2**	**Graph E**	**Function 3**
(graph)	$y = (x+1)(x+2)(x+4)$	*(graph)*	$y = -(x+1)(x+2)(x+4)$

Graph A is a reflection of Graph E over the x-axis.

How can you tell from the equation that this is a reflection?
[The negative sign in front of the function.]
If Graph A is represented by $y = f(x)$, then how can we represent Graph E?[$y = -f(x)$.]
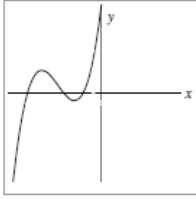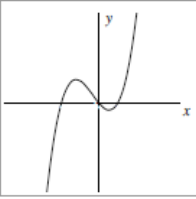
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Graph C	**Function 4**	**Graph F**	**Function 8**
(graph)	$y = (x-1)^2(x-4)$	*(graph)*	$y = -(x+1)^2(x+4)$

Graph C is a reflection of Graph F over the y-axis.

How can you tell from the equation that this is a reflection?
[Replace x by –x in the equation for Graph C and simplify,
noting that $(-x-1)^2 = (-1)(x+1)(-1)(x+1) = (x+1)^2$ and that $(-x-4) = -(x+4).$]
If Graph C is represented by $y = f(x)$, then how can we represent Graph F?[$y = f(-x)$.]

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Graph A	Function 2	Graph B	Function 1
	$y = (x+1)(x+2)(x+4)$		$y = x(x-1)(x+2)$

Graph A is a horizontal translation of Graph B.

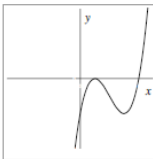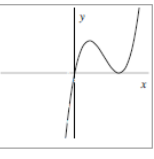*How far does Graph A have to translate horizontally to get Graph B? [+ 2 units.]*
How can you tell? [The middle root on Graph A is at $x = -2$, on Graph B it is at the origin.]
How can we transform the equation for Graph A to get the equation for Graph B?
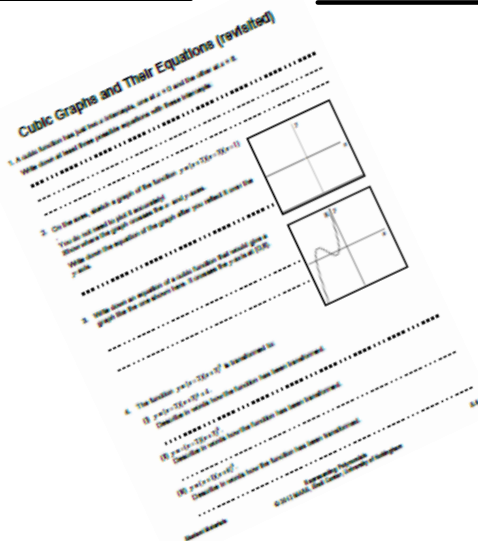*[Replace each instance of x in the equation by $x–2$.]*
If Graph A is represented by $y = f(x)$, then how can we represent Graph B? [$y = f(x–2)$.]

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Graph C	Function 4	Graph D	Function 6
	$y = (x-1)^2(x-4)$		$y = x(x-3)^2$
			Function 9
			$y = (x-1)^2(x-4)+4$

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Now Let's Assess Your Learning

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Extension:

In Groups of 2, discuss True/False Cards

Go To Answers   View Cards

Aug 27-9:32 PM

True or False?

A1

$$f(x) = x^3 - 2x^2 - 9x + 18$$

$$f(2) = 0 \Rightarrow (x+2) \text{ is a factor of } f(x)$$

A2

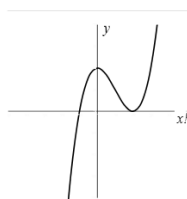$$f(x) = 3x^3 - 9x - 6$$

$$f(2) = 0 \Rightarrow (x-2) \text{ is a factor of } f(x)$$

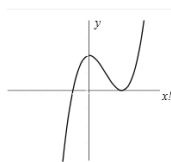## True or False?

B1
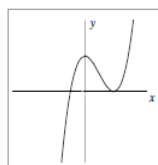
A possible equation for this graph is:
$$y = (x+1)(x \, ! \, 2)^2$$

B2

A possible equation for this graph is:
$$y = (x \, ! \, 1)(x+2)^2$$

True or False?

C1
If $f(x)$ is a cubic function
and $f(1) = 0, f(3) = 0$ and $f(4) = 0$
then $f(5) = 8$.

C2
If $f(x)$ is a cubic function
and $f(1) = 0, f(3) = 0$ and $f(4) = 0$
then $f(5)$ could be anything.

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True or False?

D1
$$f(x) = (x-2)^2(x-7)$$
$$g(x) = (x-2)^2(7-x)$$
$f(x)$ is a reflection of $g(x)$ in the y axis

D2
$$f(x) = x(x-2)^2$$
$$g(x) = -x(-x-2)^2$$
$f(x)$ is a reflection of $g(x)$ in the y axis

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True or False?

E1 $f(x) = g(x+2)$!

$g(x)$ is a translation

of $f(x)$ parallel to the x axis.

E2 $f(x) = g(x) + 2$!

$g(x)$ is a translation

of $f(x)$ parallel to the x axis.

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Extension activity: Statements to Discuss: True or False?

| A1	A2
$f(x) = x^3 - 2x^2 - 9x + 18$ $f(2) = 0 \Rightarrow (x+2)$ is a factor of $f(x)$	$f(x) = 3x^3 - 9x - 6$ $f(2) = 0 \Rightarrow (x-2)$ is a factor of $f(x)$

A1 is false; A2 is true.

If $f(2)=0$, $(x-2)$ *must* be a factor of $f(x)$.

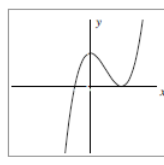| B1	B2
A possible equation for this graph is: $y = (x+1)(x-2)^2$	A possible equation for this graph is: $y = (x-1)(x+2)^2$

B1 is true; B2 is false.

There are also other possible equations for the function, such as $y = 2(x+1)(x-2)^2$.

B2 is clearly false as the repeated root must be negative.

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C1	C2
If $f(x)$ is a cubic function and $f(1) = 0$, $f(3) = 0$ and $f(4) = 0$, then $f(5) = 8$.	If $f(x)$ is a cubic function and $f(1) = 0$, $f(3) = 0$ and $f(4) = 0$, then $f(5)$ could be anything.

C1 is false; C2 is true.

The conditions mean that the function must take the form:

$f(x) = A(x-1)(x-3)(x-4)$, where A is constant.

Now $f(5) = 8A$, *so it can take any value*.

D1	D2
$f(x) = (x-2)^2(x-7)$ $g(x) = (x-2)^2(7-x)$ $f(x)$ is a reflection of $g(x)$ over the y axis	$f(x) = x(x-2)^2$ $g(x) = -x(-x-2)^2$ $f(x)$ is a reflection of $g(x)$ over the y axis

D1 is false; D2 is true.

D1 is a reflection over the x-axis.

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| E1	E2
$f(x) = g(x+2) \Rightarrow$ $g(x)$ is a horizontal translation of $f(x)$.	$f(x) = g(x) + 2 \Rightarrow$ $g(x)$ is a horizontal translation of $f(x)$.

E1 is true; E2 is false

E2 is a vertical translation.

Aug 27-9:42 PM

Extension activity: Statements to Discuss: True or False?

A1	A2
$f(x) = x^3 - 2x^2 - 9x + 18$ $f(2) = 0 \Rightarrow (x+2)$ is a factor of $f(x)$	$f(x) = 3x^3 - 9x - 6$ $f(2) = 0 \Rightarrow (x-2)$ is a factor of $f(x)$

A1 is false; A2 is true.
If $f(2)=0$, $(x-2)$ *must* be a factor of $f(x)$.

B1	B2
A possible equation for this graph is: $y = (x+1)(x-2)^2$	A possible equation for this graph is: $y = (x-1)(x+2)^2$

B1 is true; B2 is false.
There are also other possible equations for the function, such as $y = 2(x+1)(x-2)^2$.

B2 is clearly false as the repeated root must be negative.

C1	C2
If $f(x)$ is a cubic function and $f(1) = 0$, $f(3) = 0$ and $f(4) = 0$, then $f(5) = 8$.	If $f(x)$ is a cubic function and $f(1) = 0$, $f(3) = 0$ and $f(4) = 0$, then $f(5)$ could be anything.

C1 is false; C2 is true.

The conditions mean that the function must take the form:

$f(x) = A(x-1)(x-3)(x-4)$, where A is constant.

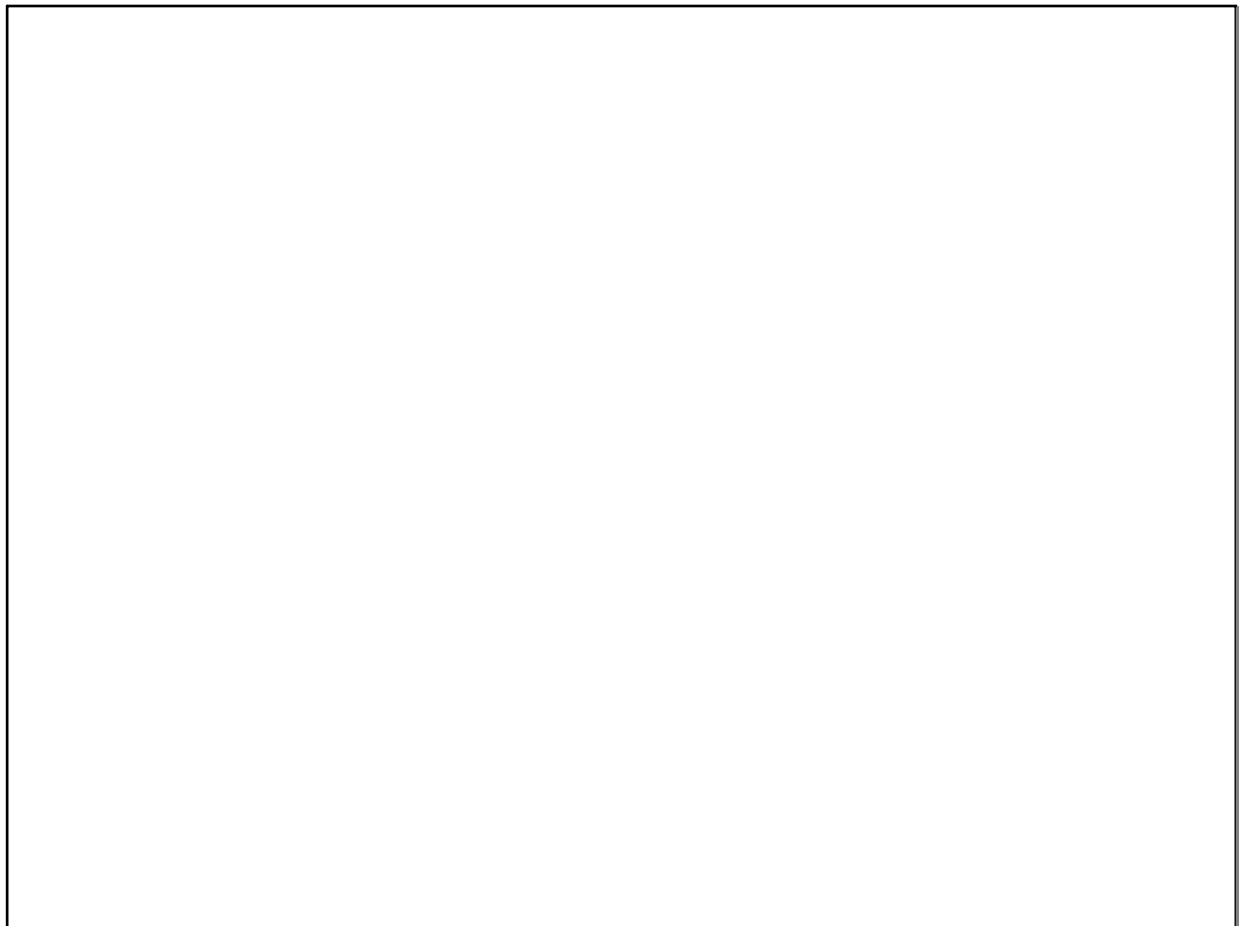Now $f(5) = 8A$, *so it can take any value.*

D1	D2
$f(x) = (x-2)^2(x-7)$ $g(x) = (x-2)^2(7-x)$ $f(x)$ is a reflection of $g(x)$ over the y axis	$f(x) = x(x-2)^2$ $g(x) = -x(-x-2)^2$ $f(x)$ is a reflection of $g(x)$ over the y axis

D1 is false; D2 is true.

D1 is a reflection over the x-axis.

E1	E2
$f(x) = g(x+2) \Rightarrow$ $g(x)$ is a horizontal translation of $f(x)$.	$f(x) = g(x)+2 \Rightarrow$ $g(x)$ is a horizontal translation of $f(x)$.

E1 is true; E2 is false

E2 is a vertical translation.

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Sep 14-5:17 PM