Exponential Functions

$f(x) = 3^x$ $f(x) = .6^x$ $f(x) = 2 \cdot \left(\frac{1}{4}\right)^x$ $f(x) = \left(\frac{13}{2}\right)^x$

When b>1, the function represents **exponential growth**.

When 0<b<1, the functions represents **exponential decay.**

Graph each of the above exponential equations and identify the y-intercept.

1. The population of the United States was 248,718,301 in 1990 and was predicted to grow at a rate of about 8% per decade.
   a. Write the expression for the population *n* decades after 1990.

   b. Predict the population, to the nearest hundred thousand, for the year 2020.

2. A certain medication is eliminated from the bloodstream at a rate of 12% per hour. The medication reaches a peak level in the bloodstream at 40 milligrams.
   a. Write the expression for the amount of medication in the blood after n hours.

   b. Predict the amount, to the nearest tenth of a milligram, the amount of medication remaining 8 hours after the peak level.

3. A virus contains bacteria that grows at a rate of 2% per year. Presently the virus contains 6000 bacteria. Find the number of bacteria 6 years from now.

Logarithmic Functions

Logarithms are used to find unknown exponents in exponential models.

$4^3 = 64$ $\longleftrightarrow$ $\log_4 64 = 3$

exponential form logarithmic form

Write each equation in logarithmic form.

1. $5^4 = 625$
2. $6^{-2} = \frac{1}{36}$
3. $\left(\frac{1}{7}\right)^{-3} = 343$
4. $3375^{\frac{1}{3}} = 15$

Write each equation in exponential form.

1. $\log_{12} 144 = 2$
2. $\log_{3600} 60 = \frac{1}{2}$
3. $\log_{\frac{1}{5}} 625 = -4$
4. $\log_{11} \frac{1}{1331} = -3$

Evaluate each expression.

1. $\log_3 81$
2. $\log_{\frac{1}{2}} 8$
3. $\log_{16} 4$

Find the value of v in each equation.

1. $5 = \log_2 v$
2. $v = \log_{20} 400$
3. $4 = \log_v 10,000$
4. $\frac{1}{2} = \log_{25} v$

Properties of Logarithmic Functions

Product Property $\log_7(xy) = \log_7 x + \log_7 y$

Quotient Property $\log_5\left(\frac{m}{n}\right) = \log_5 m - \log_5 n$

Power Property $\log_3 w^6 = 6\log_3 w$

Write each expression as a single logarithm. Then simplify if possible.

1. $\log_9 12 + \log_9 3$

2. $\log_w m + \log_w 2n - \log_w p$

3. $\log_3 4 - \log_3 18$

4. $3\log_n 4 - 2\log_n 6$

5. $\log_{11} m + 2\log_{11} n - 5\log_{11} mn$

Change-of-Base Formula

To change from base b to base a: $\log_b x = \frac{\log_a x}{\log_a b}$

Write $\log_9 27$ as a base 3 expression.

Write $\log_8 32$ as a base 2 expression.

Compound Interest Formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = total amount of the investment
P = principal
r = annual interest rate
n = # of times interest in compounded per year
annually quarterly monthly daily

t = time in years

Examples:

1. Find the final amount of a $500 investment after 8 years at 7% interest compounded
   a. monthly b. daily

2. Find the final amount of a $2000 investment after 18 months at 9.3% interest compounded
   a. quarterly b. annually