AKS 61.F.TF.8	Prove the Pythagorean identity $(\sin A)^2 + (\cos A)^2 = 1$ and use it to find sin A, cos A, or tan A, given sin A, cos A, or tan A, and the quadrant of the angle.
What does this standard mean that a student must know, understand, and be able to do?	
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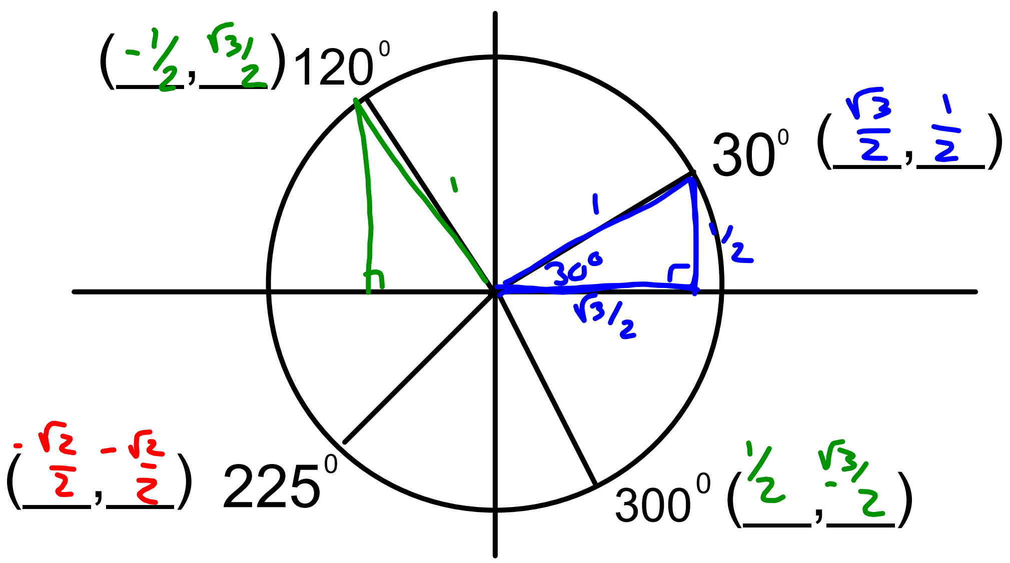
Background Knowledge/ Connections	Students should know the Pythagorean theorem and that $\tan\theta = \sin\theta/\cos\theta$.
Vocabulary	Pythagorean theorem, identity, quadrant.
Instructional	An **identity** is an equation that is true for all values of the variable for which the expressions in the equation are defined.

Unit Circle



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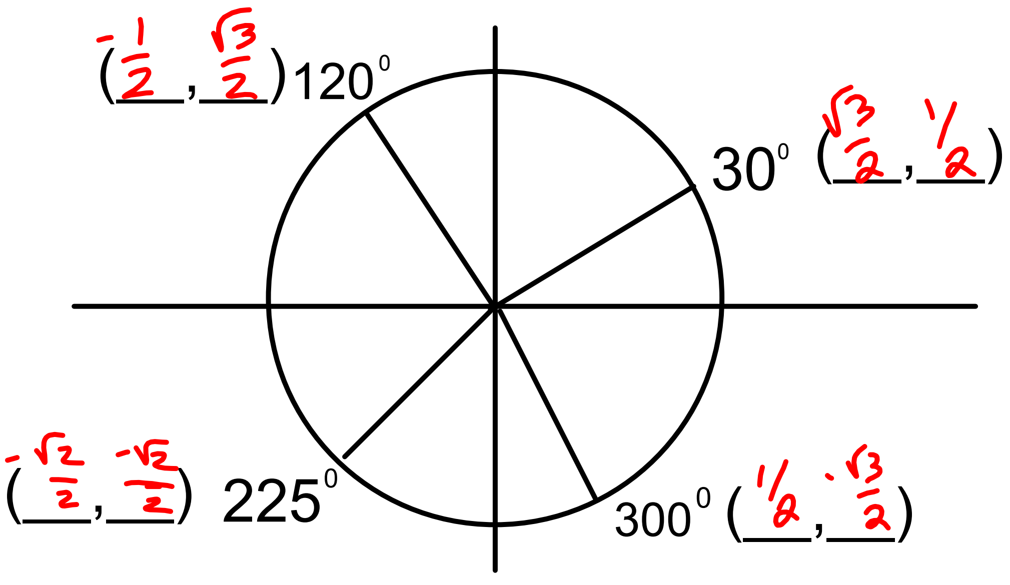
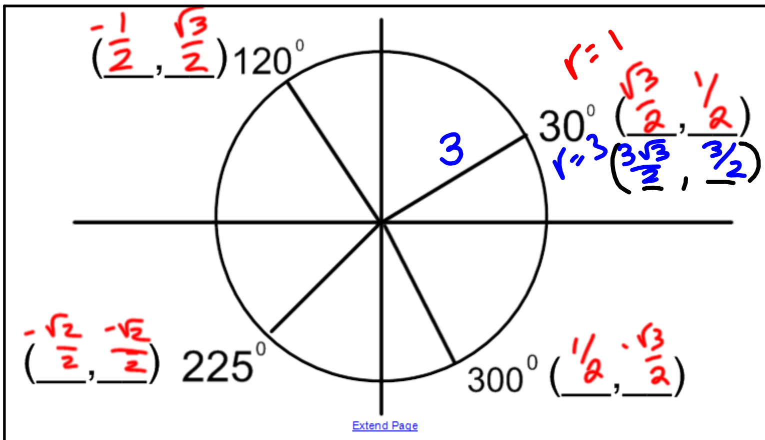
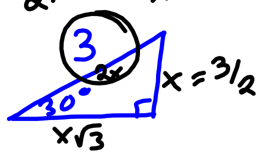
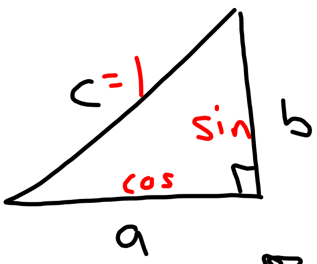




$2x=3 \quad x=3/2$



$\frac{3}{2}\cdot\sqrt{3} = \frac{3\sqrt{3}}{2}$



$a^2+b^2=c^2$

$\cos^2\theta+\sin^2\theta=1^2$

$\cos^2\theta+\sin^2\theta=1$

Given:

$\cos\theta=\frac{1}{2}$

$\left(\frac{1}{2}\right)^2 \quad \frac{1^2}{2^2}$

$\left(\frac{1}{2}\right)^2+\sin^2\theta=1$

$\frac{1}{4}+\sin^2\theta=1$

$-1/4 \qquad -1/4$

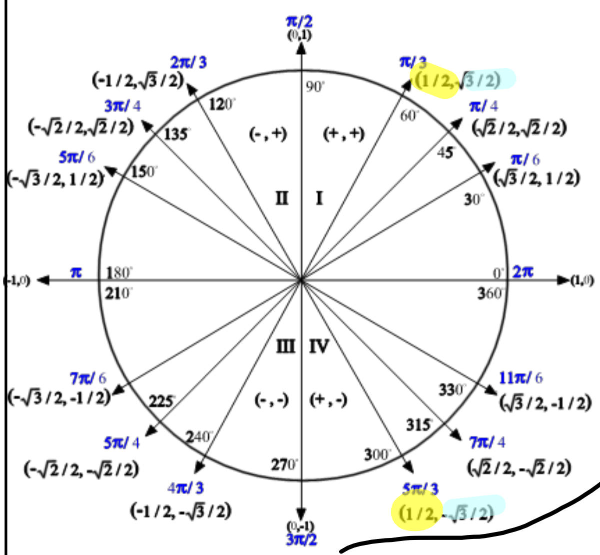
$\sin^2\theta=1-\frac{1}{4}$

$\sin^2\theta=\frac{4}{4}-\frac{1}{4}$

$\sqrt{\sin^2\theta}=\pm\sqrt{\frac{3}{4}}$

$\sin\theta=\pm\sqrt{\frac{3}{4}}$

$\sin\theta=\pm\frac{\sqrt{3}}{\sqrt{4}}=\frac{\sqrt{3}}{2}$



$\cos\theta=\frac{1}{2} \qquad \sin\theta=\pm\frac{\sqrt{3}}{2}$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

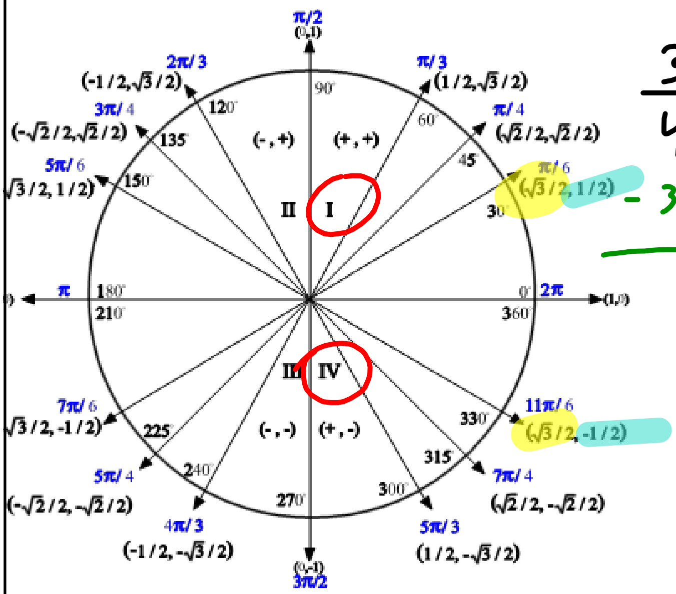
$$\cos^2\theta + \sin^2\theta = 1$$

$$\left(\frac{\sqrt{3}}{2}\right)^2 + \sin^2\theta = 1$$

$$\frac{(\sqrt{3})^2}{2^2} + \sin^2\theta = 1$$

$$\frac{3}{4} + \sin^2\theta = 1 \qquad \frac{4}{4} - \frac{3}{4} = \frac{1}{4}$$

$$-3/4 \qquad -3/4$$



$$\sqrt{\sin^2\theta} = \pm\sqrt{\frac{1}{4}}$$

$$\sin\theta = \pm\frac{\sqrt{1}}{\sqrt{4}} = \pm\frac{1}{2}$$

$$\cos\theta = \frac{\sqrt{3}}{2} \qquad \sin\theta = \pm\frac{1}{2}$$

$\cos^2\theta + \sin^2\theta = 1$

$\cos\theta = \frac{1}{5}$ $\sin\theta = ?$

$\left(\frac{1}{5}\right)^2 + \sin^2\theta = 1$

$\frac{1}{25} + \sin^2\theta = 1$ $\frac{25}{25} - \frac{1}{25}$

$\sqrt{\sin^2\theta} = \pm\sqrt{\frac{24}{25}}$

$\sin\theta = \pm\frac{\sqrt{24}}{\sqrt{25}} \pm \frac{\sqrt{24}}{5}$

$\sqrt{24}$

$\sqrt{4 \cdot 6}$

$2\sqrt{6}$

$\sin\theta = \pm\frac{\sqrt{24}}{5} = \pm\frac{2\sqrt{6}}{5}$

1st year 098
099
101
1001
2nd 201
3rd 301
4th 401

098 No credit
099
101
201
301
401

Compass test

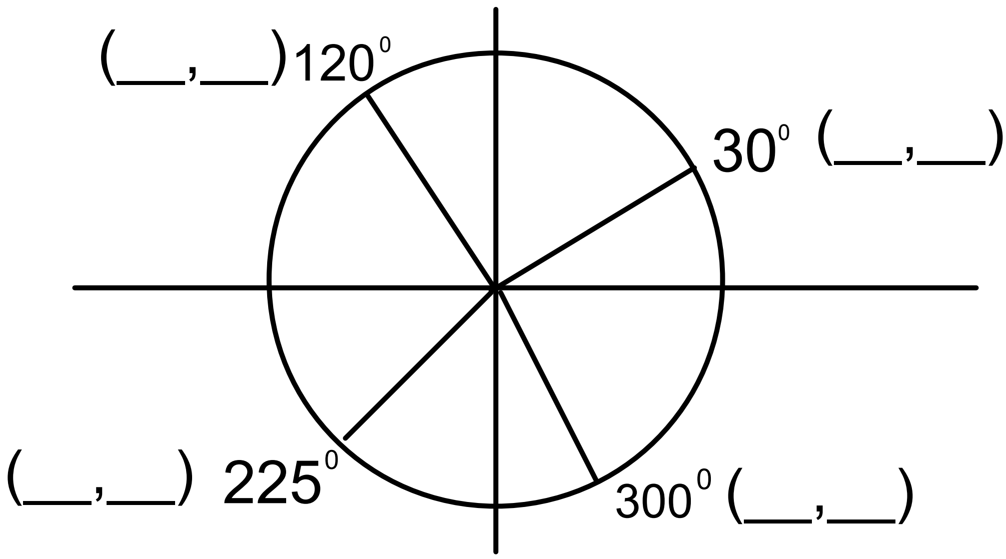
SAT ACT

precalc.

Oct 30-10:47 AM

What if the radius is not equal to 1?

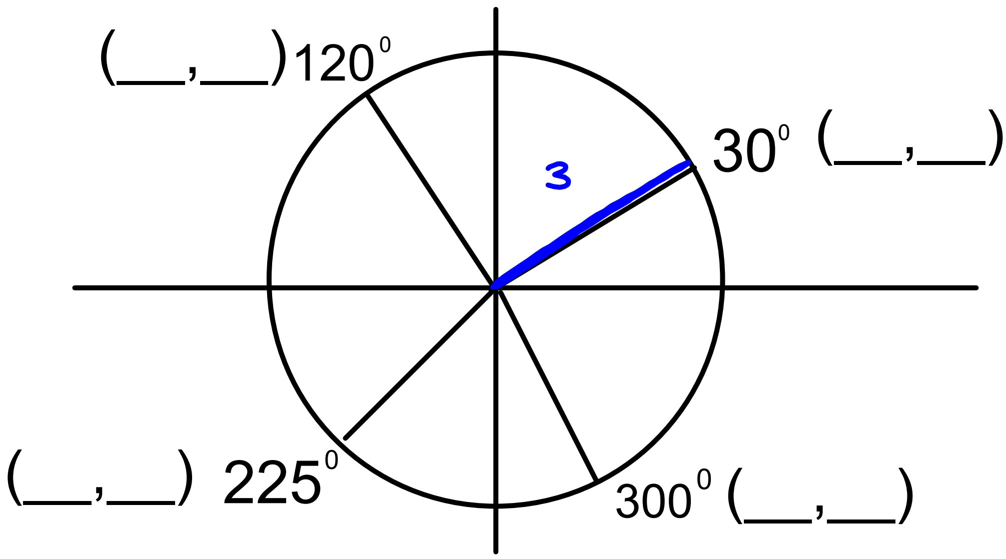
How can you find the coordinates?



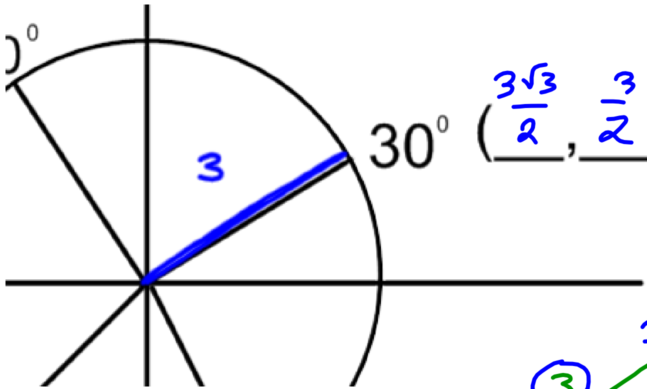
Oct 29-3:47 PM

What if the radius is not equal to 1?

How can you find the coordinates?



Oct 29-3:47 PM

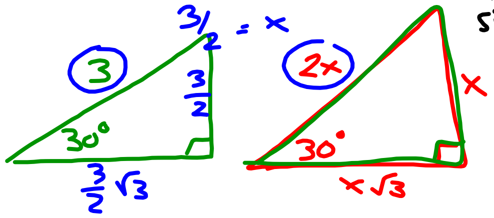


$\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

Forgot the ratio of sides.

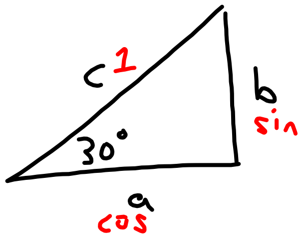
$3 = 2x$

$\frac{3}{2} = x$



$\frac{3\sqrt{3}}{2}$

$a^2 + b^2 = c^2$



$\cos^2\theta + \sin^2\theta = 1^2$

$\cos^2\theta + \sin^2\theta = 1$

$\boxed{\cos\theta} = \frac{1}{2}$

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{1}{2}\right)^2 + \sin^2\theta = 1$

$\frac{1^2}{2^2} = \frac{1}{4}$

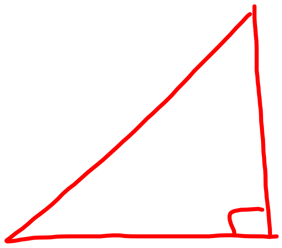
$\frac{1}{4} + \sin^2\theta = 1$

$-\frac{1}{4} \qquad -\frac{1}{4}$

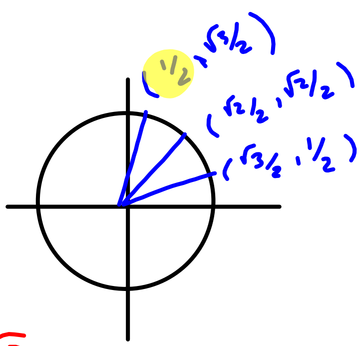
$\sqrt{\sin^2\theta} = \sqrt{\frac{3}{4}}$

$1 - \frac{1}{4} = \frac{3}{4}$

example:



$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$

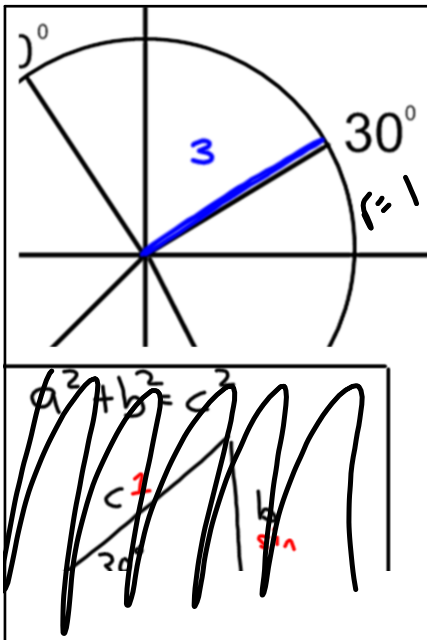


$\sin\theta = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$

$\cos\theta = \frac{1}{2} \qquad \sin\theta = \frac{\sqrt{3}}{2}$

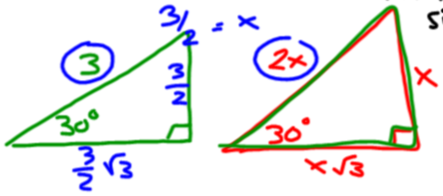
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Oct 30-7:27 AM



$3 = 2x$

$\frac{3}{2} = x$



$\frac{3\sqrt{3}}{2}$

Forgot the ratio of sides.

$$\frac{3}{1}\left(\frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}}{2}$$

$$\frac{3}{1}\left(\frac{1}{2}\right) = \frac{3}{2}$$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{\sqrt{3}}{2}\right)^2 + \sin^2\theta = 1$

$(\sqrt{3})^2$

$\sqrt{3}\cdot\sqrt{3} = 3$

$\frac{(\sqrt{3})^2}{2^2} + \sin^2\theta = 1$

$\frac{3}{4} + \sin^2\theta = 1$

$-3/4 \qquad -3/4$

$1 - \frac{3}{4}$

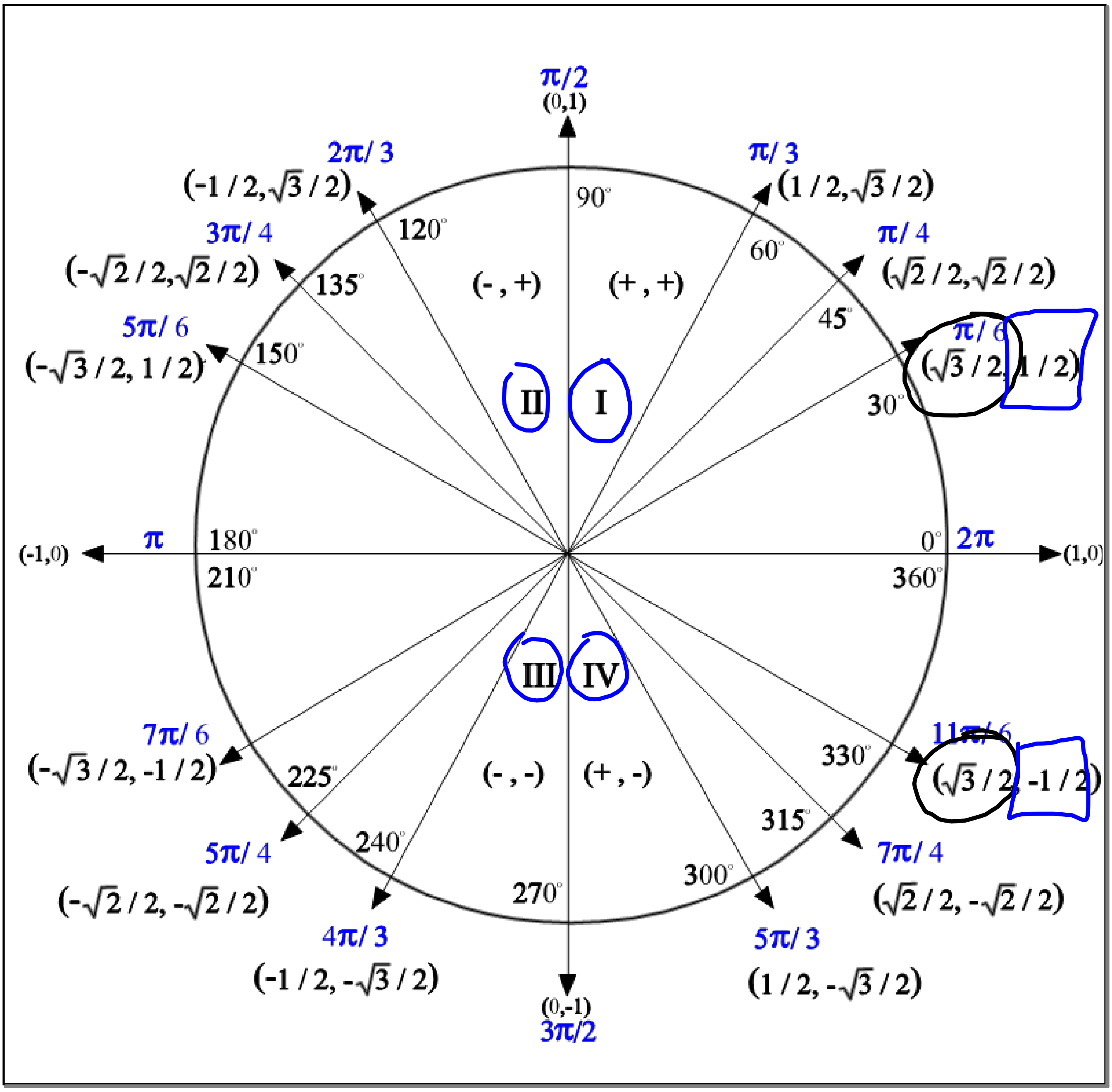
$\frac{4}{4} - \frac{3}{4} = \frac{1}{4}$

$\sin^2\theta = 1 - \frac{3}{4}$

$\sqrt{\sin^2\theta} = \pm\sqrt{\frac{1}{4}}$

$\sin\theta = \pm\frac{\sqrt{1}}{\sqrt{4}} = \pm\frac{1}{2}$

$\cos\theta = \frac{\sqrt{3}}{2} \qquad \sin\theta = \pm\frac{1}{2}$


π/2
(0,1)
2π/3
(-1/2, √3/2)
π/3
(1/2, √3/2)
3π/4
(-√2/2, √2/2)
π/4
(√2/2, √2/2)
5π/6
(-√3/2, 1/2)
π/6
(√3/2, 1/2)
90°
120°
60°
135°
45°
150°
30°
(- , +)
(+ , +)
II
I
π
(-1,0)
180°
210°
0°
2π
360°
(1,0)
III
IV
(- , -)
(+ , -)
7π/6
(-√3/2, -1/2)
11π/6
(√3/2, -1/2)
225°
330°
315°
240°
300°
270°
5π/4
(-√2/2, -√2/2)
7π/4
(√2/2, -√2/2)
4π/3
(-1/2, -√3/2)
5π/3
(1/2, -√3/2)
(0,-1)
3π/2

$\left(\overset{\cos}{?} , \overset{\sin}{\frac{1}{5}} \right)$

$\sin\theta = \frac{1}{5}$

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{1}{5}\right)^2 = \frac{1^2}{5^2} = \frac{1}{25}$

$\cos^2\theta + \left(\frac{1}{5}\right)^2 = 1$

$\cos^2\theta + \frac{1}{25} = 1$

$\quad - \frac{1}{25} \quad - \frac{1}{25}$

$\frac{1}{1} - \frac{1}{25}$

$\frac{25}{25} - \frac{1}{25} = \frac{24}{25}$

$\sqrt{\cos^2\theta} = \sqrt{\frac{24}{25}}$

$\cos\theta = \frac{\pm\sqrt{24}}{\sqrt{25}}$

$\sqrt{24}$

$\sqrt{4 \cdot 6}$

$2\sqrt{6}$

$\cos\theta = \frac{\pm\sqrt{24}}{5} = \frac{\pm 2\sqrt{6}}{5}$

097 097
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Oct 30-8:03 AM

Oct 30-10:10 AM

What if we use the Pythogrean Theorm?

$$a^2 + b^2 = c^2$$

sin + cos = radius

Oct 29-3:48 PM

1. There are several important identities in trigonometry. Let's see if we can discover and make sense of one of these identities. Other identities will be explored in a later unit.

   a. Randomly list four real numbers in the first column of the table. Use your calculator to complete the table.

Comment: You may want to encourage students to list at least one of the special real numbers such as $\frac{\pi}{4}$, $\frac{7\pi}{6}$, π, etc for which the exact values of sine and cosine are known.
Be sure students "square the sine of t" rather than "taking the sine of t^2." Show them how to enter this in the calculator correctly.

Real number t	$(\sin t)^2$	$(\cos t)^2$	$(\sin t)^2 + (\cos t)^2$

   b. What observations can you make about the values in your table?

   Ask all groups in the class to report on their observations.

   c. Based on your observations, what statement can you make about real number t?

   Use the data from the class to conjecture that $(\sin t)^2 + (\cos t)^2 = 1$ for any real number t.
   Once students make the conjecture, ask if the equation is an identity. Be sure students understand that even though we showed the equation was true for many values of t, we still have not shown the equation is true for ALL real numbers t—and this is impossible to do by substitution. We need to use deductive reasoning to prove it is an identity.

   d. Use technology to graph $y = (\sin x)^2 + (\cos x)^2$. What do you observe?

Oct 29-3:42 PM

The graph of $y = (\sin x)^2 + (\cos x)^2$ *is the horizontal line* $y = 1$ *since* $(\sin x)^2 + (\cos x)^2 = 1$

e. Using what you know about the equation of the unit circle, the way we pair real numbers to points on the unit circle, and how sine and cosine are defined in terms of the unit circle, give a viable argument for why $(\sin t)^2 + (\cos t)^2 = 1$ for all real numbers t.

The equation of the unit circle is $x^2 + y^2 = 1$. *Because of the way we pair real numbers to points on the unit circle, we know for real number t there is an angle in standard position of t radians that intercepts an arc t on the unit circle that has a terminal point P (x, y). The sine and cosine of real number t is defined in terms of the terminal point of t on the unit circle so we can write the coordinates of point P as (cos t, sin t). Thus, if we replace x and y in* $x^2 + y^2 = 1$ *with cos t and sin t, respectively, we get* $(\cos t)^2 + (\sin t)^2 = 1$ *or, of course,* $(\sin t)^2 + (\cos t)^2 = 1$.

EXAMPLE: The identity $(\sin t)^2 + (\cos t)^2 = 1$ is referred to as a Pythagorean Identity. Let's see how it can be used to answer the following question:

Suppose $\cos t = \frac{1}{2}$. What is the exact value of $\sin t$ if t terminates in the first quadrant?

We can find the answer with the following strategy:

For any real number *t*, $(\sin t)^2 + (\cos t)^2 = 1$.

Since cost t = ½ we have $(\sin t)^2 + \left(\frac{1}{2}\right)^2 = 1$.

Therefore, $(\sin t)^2 = \frac{3}{4}$ Why?

$\sin t = \pm\frac{\sqrt{3}}{2}$ Why?

Since t terminates in Quadrant I, then *sin t > 0*. Therefore, $\sin t = \frac{\sqrt{3}}{2}$.

Now that we know $\sin t = \frac{\sqrt{3}}{2}$ and $\cos t = \frac{1}{2}$, what is the value of $\tan t$?

$$\tan t = \frac{\sin t}{\cos t} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

2. Use the strategy outlined in the above example to answer the following questions.

a. $\sin t = \frac{1}{3}$ and t terminates in Quadrant 2. What are the exact values of $\cos t$ and $\tan t$?

$\cos t = -\sqrt{\frac{8}{9}} = \frac{-2\sqrt{2}}{3}$ $\tan t = -\frac{1}{2\sqrt{2}}$ *or* $-\frac{\sqrt{2}}{4}$

b. $\cos t = \frac{-\sqrt{3}}{5}$ and t is in the 3rd quadrant. What are the exact values of $\sin t$ and $\tan t$?

$\sin t = -\frac{\sqrt{22}}{5}$ $\tan t = \sqrt{\frac{22}{3}}$ or $\frac{\sqrt{66}}{3}$

Students frequently confuse quadrants 1 and 2 and quadrants 3 and 4. Some students learn the quadrant signs by "All Students Take Calculus" indicating which sin, cos, or tan are positive.