

6.2 Notes – Confidence Intervals for the Mean (Small Samples)

I. The t -Distribution

In many real-life situations, the population standard deviation is unknown. If the random variable is normally distributed (or approximately normal) the sampling distribution for \bar{x} is a t -distribution.

\bar{x} sample mean

Formula for t -distribution by _____.

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Critical values of t are denoted

Properties of the t -distribution:

- 1.
2. **degrees of freedom (df) – (use table)**
- 3.
- 4.
- 5.

Example 1 – use the table:

- a. Find the critical value t_c , for a 90% confidence when the sample size is 22.
- b. Find the critical value, t_c , for a 95% confidence when the sample size is 15.
- c. Find the critical value, t_c , for an 99% confidence when the sample size is 28.

II. Confidence Intervals and t -Distributions

Example 2:

You randomly select 16 restaurants and measure the temperature of the coffee sold at each. The sample mean temperature is 162°F with a sample standard deviation of 10°F. Find the 90% confidence interval for the mean temperature.

Example 1 - use the table:

- a. Find the critical value t_c , for a 90% confidence when the sample size is 22.
- b. Find the critical value, t_c , for a 95% confidence when the sample size is 15.
- c. Find the critical value, t_c , for an 99% confidence when the sample size is 28.

a) $t_c = ?$

Given
 90% confidence
 $n = 22$


Formula
 $d.f. = n - 1$

Steps:
 ① $d.f. \Rightarrow n = 22$
 $22 - 1$
 $= 21$

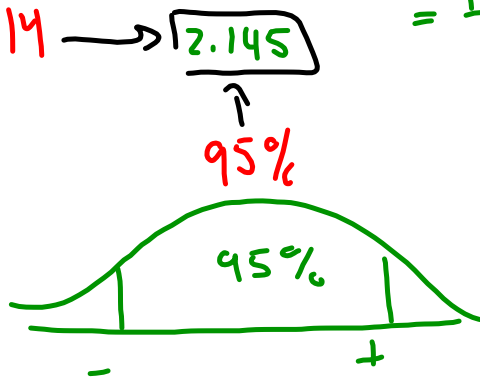
$t_c = 1.721$

② t-table
 * left side
 d.f.
 21
 * bottom find
 90% confidence
 1.721

- b. Find the critical value, t_c , for a 95% confidence when the sample size is 15.
 c. Find the critical value, t_c , for an 99% confidence when the sample size is 28.

b) steps $n=15$
 ① d.f. $n-1 \Rightarrow 15-1$
 ② use table 
 * d.f. = 14
 * confidence = 95%

d.f. $t_c = 2.145$
 $= \pm 2.145$



c)
 d.f. = 27
 99%
 $t_c = \pm 2.771$

Properties of the t -distribution:

1. t -distribution is bell-shaped and symmetric about the mean.

2. degrees of freedom (df) -

$$df = n - 1$$

(use table)

3. total area = 100% or 1.00

4. the mean, median, and mode of the t -distribution are equal to zero

5. as the degrees of freedom increase the distribution is very close to a normal distribution ($d.f. > 30$)

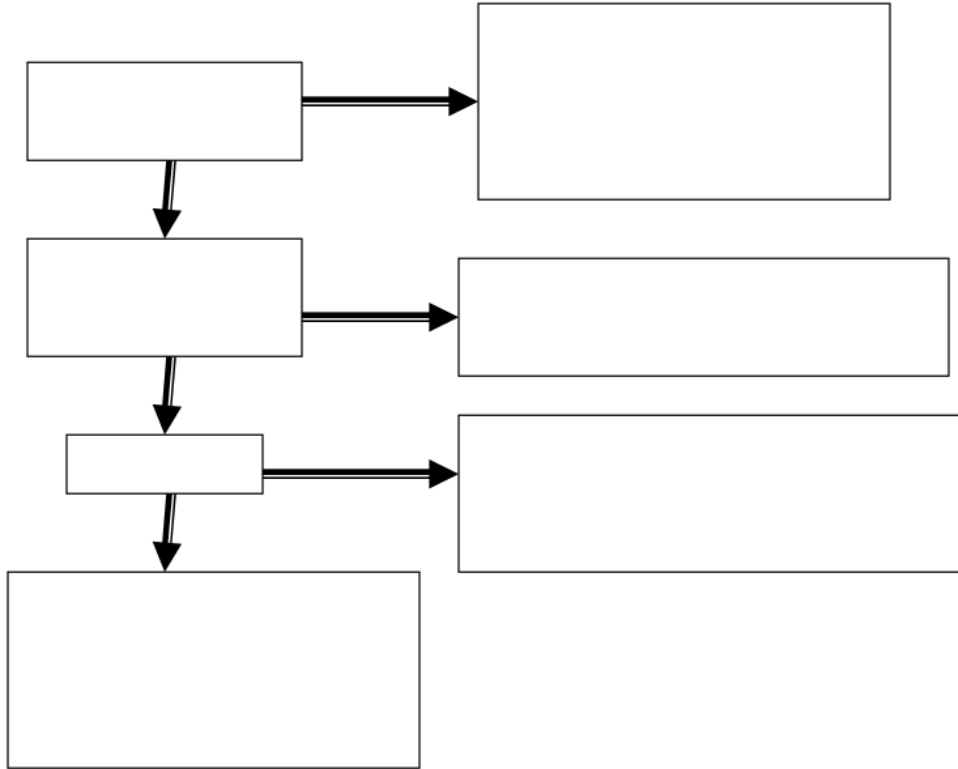
$$df = n - 1$$

* you use a t -distribution to estimate a population mean

6.2 Notes – Confidence Intervals for the Mean

(Small Samples)

How do you know when to use a normal distribution or a t -distribution to construct a confidence interval?



Example 3:

You randomly select 18 adult male athletes and measure the resting heart rate of each. The sample mean heart rate is 64 beats per minutes with a sample standard deviation of 2.5 beats per minute. Assuming heart rates are normally distributed, should you use the normal distribution or the t -distribution, or neither to construct a 90% confidence interval for the mean heart rate? Find the interval, if possible.

Assignment: new: pgs 330 – 331/1, 2, 13, 15, 17, 20, 21

Normal Distribution or t-distribution?

To work the following problems, would you use a normal distribution (z) or a t-distribution?

1. A city spokesperson claims that the mean response time for arrival of a fire truck at a fire is 12 minutes. A newspaper reporter suspects that the response time is actually longer and runs a test by examining the records of 64 fire emergency situations. If the sample mean is 13.1 minutes with a standard deviation of 6 minutes, find a 95% confidence interval.
2. In crash tests of 15 Honda Odyssey minivans, collision repair costs are found to have a distribution that is roughly normal, with a mean of \$1786 and a standard deviation of \$937 (based on data from the Highway Loss Data Institute). Construct a 99% confidence interval for the mean repair costs in all such vehicle collisions.
3. In a time-use study, 20 randomly selected managers were found to spend a mean of 2.4 hours each day on paperwork. The standard deviation of the 20 scores is 1.3 hours (based on data from Adia Personnel Services). Also, the sample data appear to take on the shape of a normal distribution. Construct a 95% confidence interval for the mean of all deliveries.
4. In a study of the use of hypnosis to relieve pain, sensory ratings were measured, with the results listed below. Use these sample data to find the standard error of the mean.
8.8 6.6 8.4 6.5 8.4 7.0 9.0 10.3
8.7 11.3 8.1 5.2 6.3 8.7 6.2 7.9
5. The Bank of New England is concerned with the amount of debt being accrued by customers using its credit cards. The Board of Directors voted to institute an expensive monitoring system if the mean debt for all of the bank's customers is greater than \$2000. The bank randomly selected 50 credit-card holders and determined that the mean amount they charged was \$2177 with a standard deviation of \$1257. Find a 90% confidence interval for the true mean credit card charges.
6. The population mean weight of laboratory mice is 25.0 g with a standard deviation of 1.35 g. A sample of 25 mice has a mean of 24.7 g. We can assume the population of mice is approximately normally distributed. Find a 99% confidence interval for the true mean weight of the mice.

