

Prove the following identities.

1. $\tan(\theta)\cot(\theta)=1$

2. $\frac{\cos(\theta)}{1-\sin^2(\theta)}=\sec(\theta)$

3. $\csc^2(\phi)-\csc^2(\phi)\cos^2(\phi)=1$

4. $\frac{\cos(x)\sin(x)}{(1-\sin^2(x))}=\tan(x)$

5. $\frac{\tan(\theta)\cos(\theta)\csc(\theta)}{(\sec(\theta)-\tan(\theta))(\sec(\theta)+\tan(\theta))}=1$

6. $\frac{(\sec(\theta)+1)(\sec(\theta)-1)}{\sin^2(\theta)}=\sec^2(\theta)$

7. $\tan x + \cot x = \sec x \csc x$

8. $\sin\theta\cos\theta\tan\theta + \cos^2\theta = 1$

9. $\frac{1-\sin^2x}{1-\cos^2x}=\cot^2x$

10. $\frac{\sin x \cos x}{1-\sin^2x}=\tan x$

11. $(1-\cos x)(1+\sec x)=\sin x \tan x$

12. $\sec^2x - \tan x \cot x = \tan^2x$

13. $7\sec\theta + 5\csc\theta = \frac{7\sin\theta + 5\cos\theta}{\sin\theta\cos\theta}$

Sum and Difference

Find the exact value if $0 < x < \frac{\pi}{2}$ and $0 < y < \frac{\pi}{2}$

1. $\sec(x-y)$ if $\tan x = \frac{7}{6}$ and $\csc y = \frac{8}{5}$

2. $\cos(x-y)$ if $\cos x = \frac{5}{13}$ and $\cos y = \frac{4}{5}$

3. $\tan(x-y)$ if $\sin x = \frac{8}{17}$ and $\cos y = \frac{3}{5}$

4. $\cos(x-y)$ if $\cos x = \frac{3}{5}$ and $\cos y = \frac{4}{5}$

5. $\cos(x+y)$ if $\sin x = \frac{5}{13}$ and $\sin y = \frac{4}{5}$

6. $\sin(x - y)$ if $\cos x = \frac{8}{17}$ and $\cos y = \frac{3}{5}$

7. $\tan(x - y)$ if $\csc x = \frac{13}{5}$ and $\cot y = \frac{4}{3}$

Verify that each equation is an identity

11. $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$

12. $\cos(180^\circ + x) = -\cos x$