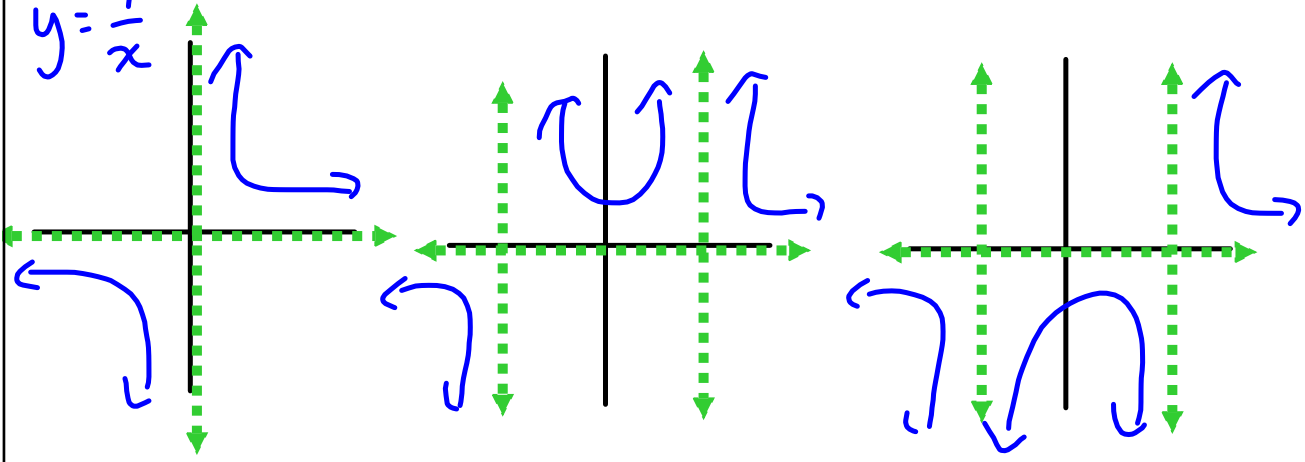
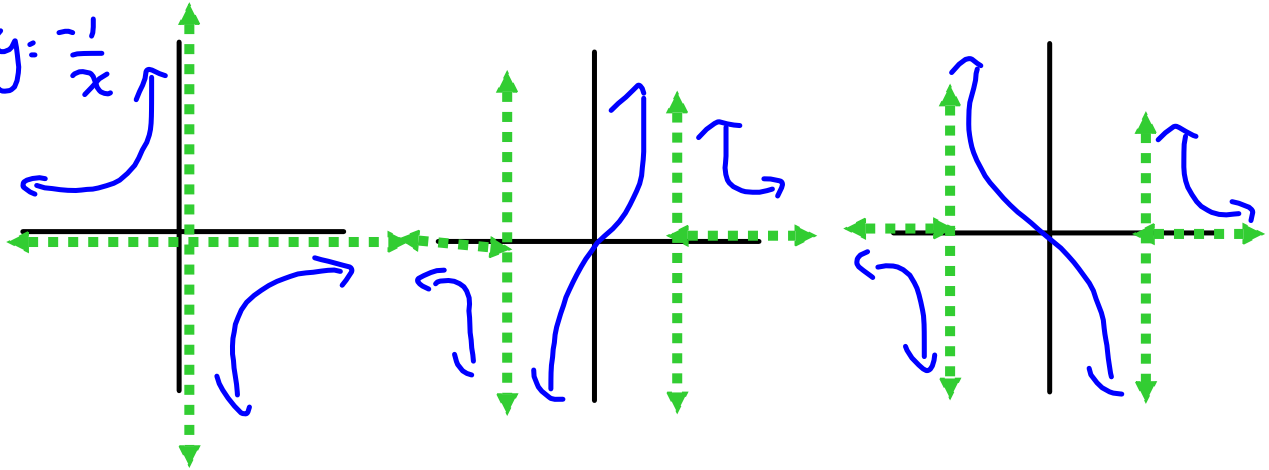


What can the graphs look like?

$$y = \frac{1}{x}$$



$$y = -\frac{1}{x}$$



$$f(x) = \frac{x^2 + 4x}{4x^2 - 16} \leftarrow \begin{matrix} \text{GCF} \\ \text{Perfect Square} \end{matrix} \quad \frac{x(x+4)}{\cancel{(2x-1)}\cancel{(2x+4)}}$$

$$\frac{x(x+4)}{4(x-2)(x+2)} \quad \begin{matrix} \text{GCF} \\ 4(x^2-4) \\ 4(x-2)(x+2) \end{matrix}$$

Cancel out \Rightarrow nothing \Rightarrow no holes

denom = 0 $4(x-2)(x+2) = 0 \Rightarrow$ VA "vertical asymptote"

$$\frac{\cancel{4}(x-2)(x+2) = 0}{\cancel{4}} \quad \frac{1}{4} \quad x = -2 \quad x = 2$$

$$(x-2)(x+2) = 0$$

$$\begin{matrix} x-2=0 & x+2=0 \\ x=2 & x=-2 \end{matrix}$$

Horizontal Asymptote Rules

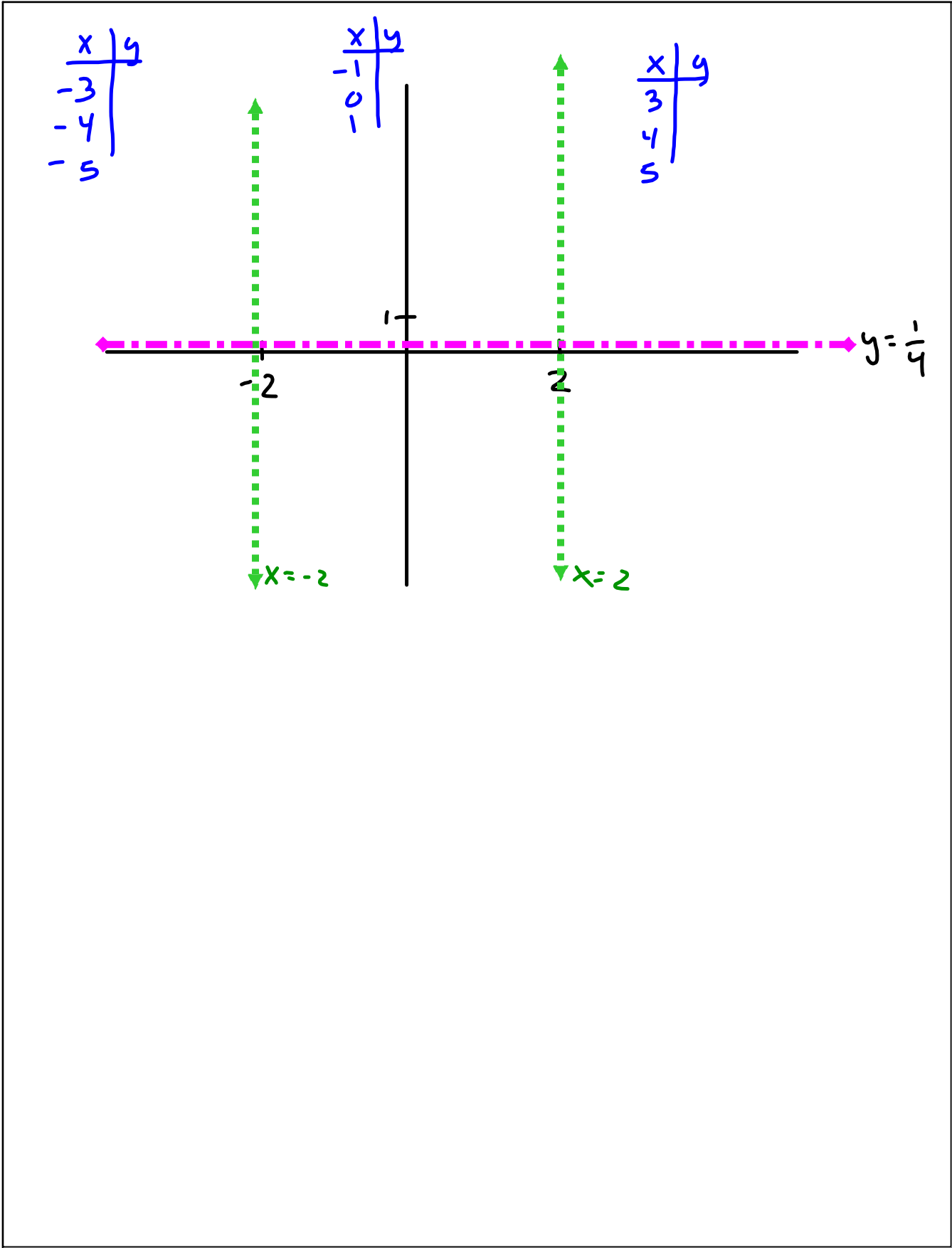
1) largest degree on top $\frac{x^3 \dots}{x \dots}$
Slant

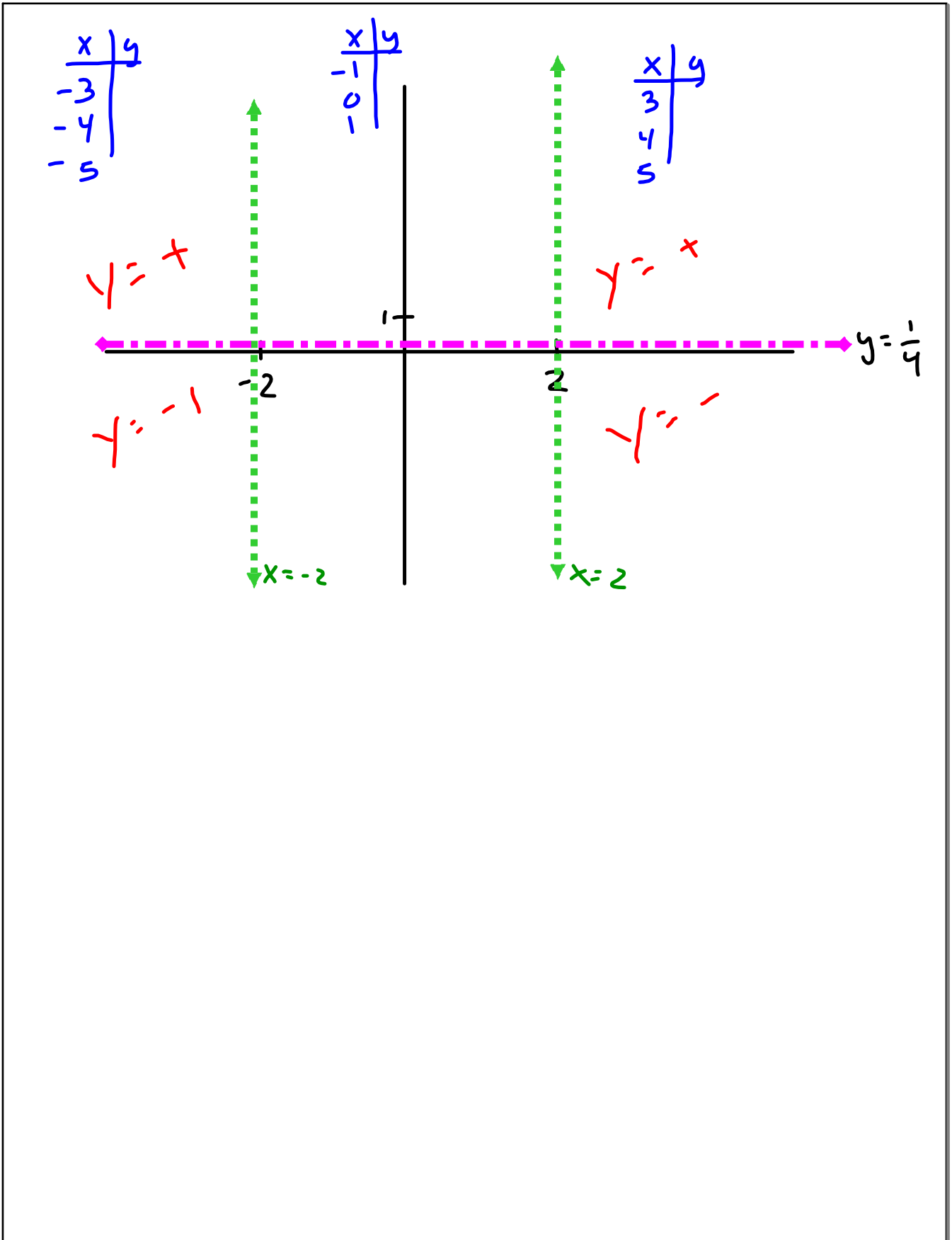
2) largest degree on bottom $\frac{x \dots}{x^2 \dots}$
 $y = 0$

3) same degree top & bottom
coefficients of highest degrees $y = \frac{3}{5}$ $\boxed{\frac{3x^2 \dots}{5x^2 \dots}}$

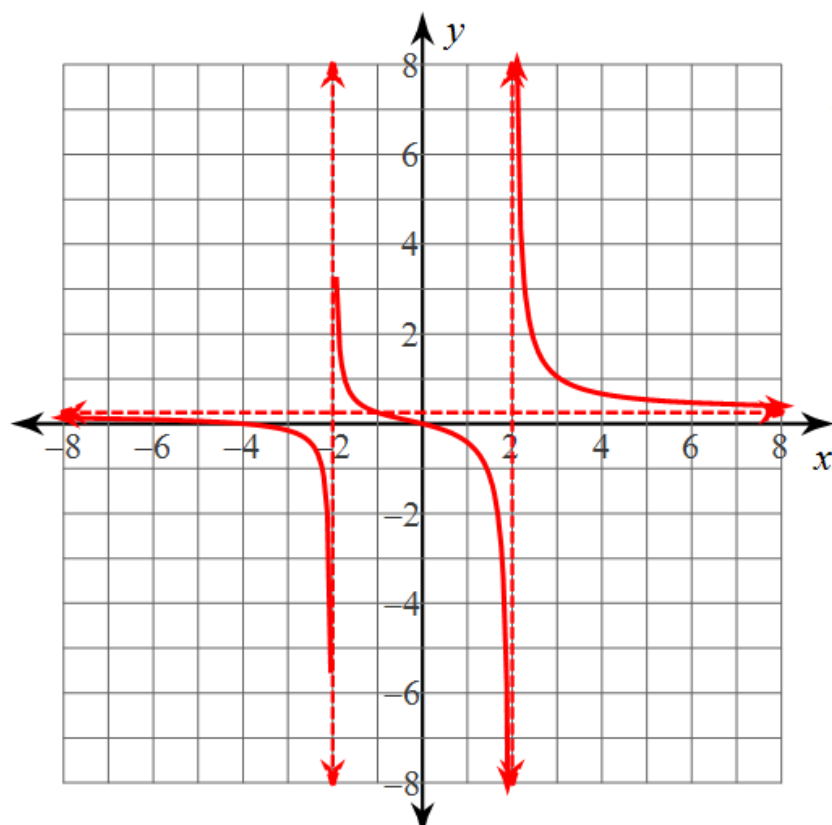
$$\frac{x^2}{4x^2}$$

$$y = \frac{1}{4}$$





$$2) f(x) = \frac{x^2 + 4x}{4x^2 - 16}$$

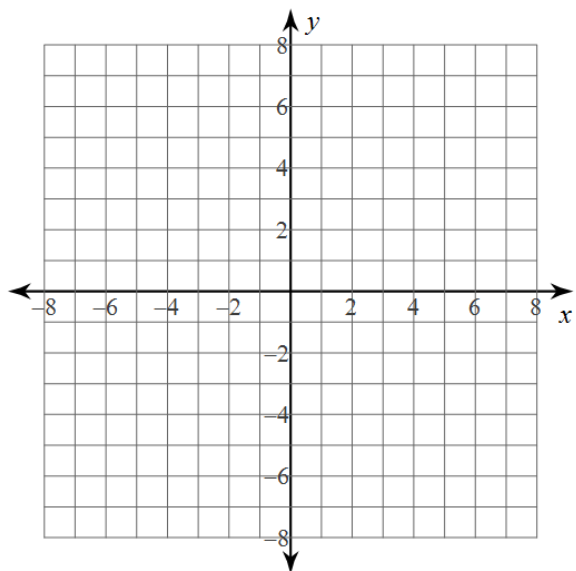


Vertical Asym.: $x = 2, x = -2$

Holes: None

Horz. Asym.: $y = \frac{1}{4}$

$$3) f(x) = \frac{1}{4x^2 - 36}$$



VA denom=0
Holes cancel
H/A Rules

$$3) f(x) = \frac{1}{4x^2 - 36}$$

$$4(x^2 - 9)$$

$$4(x-3)(x+3)$$

$$4(x-3)(x+3) = 0$$

$$x-3=0 \quad x+3=0$$

$$x=3 \quad x=-3$$

Vertical Asym.: $x = 3, x = -3$

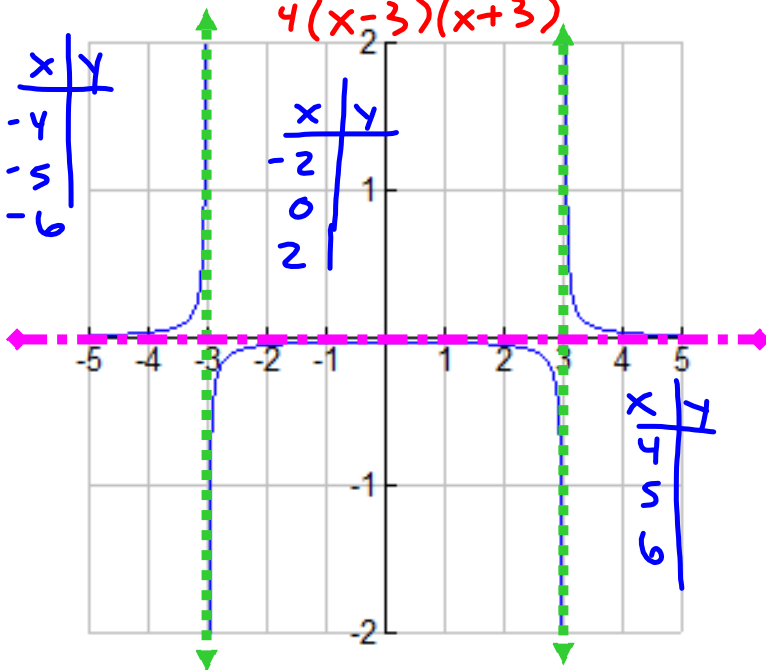
Holes: None

Horz. Asym.: $y = 0$

Rules

Highest degree
on bottom

$$y = 0 \quad \frac{1}{4x^2 - 36}$$



-2.

What can the graphs look like?

The diagram illustrates the graphs of rational functions with two vertical asymptotes. It is divided into two rows, each showing a coordinate system with a horizontal x-axis and a vertical y-axis. The horizontal asymptote is represented by a dashed green line, and the vertical asymptotes are represented by solid black lines. The regions between the asymptotes are labeled with arrows indicating the function's behavior.

Top Row: $y = \frac{1}{x}$

- Left of the first vertical asymptote:** A red arrow points up and to the right, indicating the function approaches positive infinity as x approaches the asymptote from the left.
- Between the two vertical asymptotes:** A blue U-shaped curve opens upwards, indicating the function is positive in this region.
- Right of the second vertical asymptote:** A purple curve passes through the x-axis, indicating the function is negative in this region.

Bottom Row: $y = -\frac{1}{x}$

- Left of the first vertical asymptote:** A red arrow points down and to the right, indicating the function approaches negative infinity as x approaches the asymptote from the left.
- Between the two vertical asymptotes:** A blue inverted U-shaped curve opens downwards, indicating the function is negative in this region.
- Right of the second vertical asymptote:** A purple curve passes through the x-axis, indicating the function is positive in this region.

$$f(x) = \frac{x^2 + 4x}{4x^2 - 16}$$

1. Simplify
2. Cancel out -- find the holes
3. denominator = 0 -- vertical asymptotes
4. highest degree -- horizontal asymptote

$$f(x) = \frac{x^2 + 4x}{4x^2 - 16}$$

1. Simplify $\frac{x(x+4)}{4(x^2-4)}$ $\frac{x(x+4)}{4(x-2)(x+2)}$

2. Cancel out -- find the holes

$$\frac{x(x+4)}{4(x-2)(x+2)}$$

Nothing cancels out
No Holes

3. denominator = 0 -- vertical asymptotes

$$4(x-2)(x+2) = 0 \quad x = -2 \quad x = 2$$

$$\cancel{4}(x-2)(x+2) = \frac{0}{\cancel{4}}$$

$$(x-2)(x+2) = 0$$

$$x-2=0 \quad x+2=0$$

$$x=2 \quad x=-2$$

4. highest degree -- horizontal asymptote

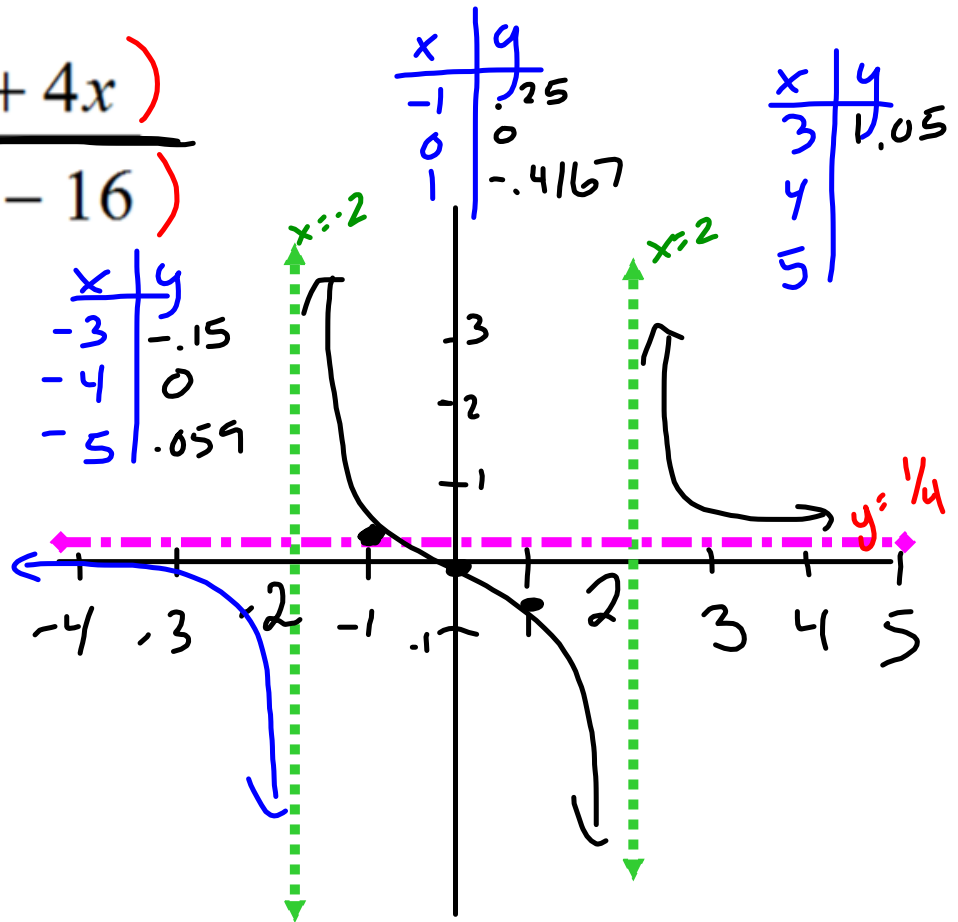
Rules :
 highest degree is top slant
 bottom $x=0$
 same coefficients of highest degree

$$y = \frac{x(x+4)}{4(x-2)(x+2)} = \frac{x^2 + 4x}{4x^2 - 16}$$

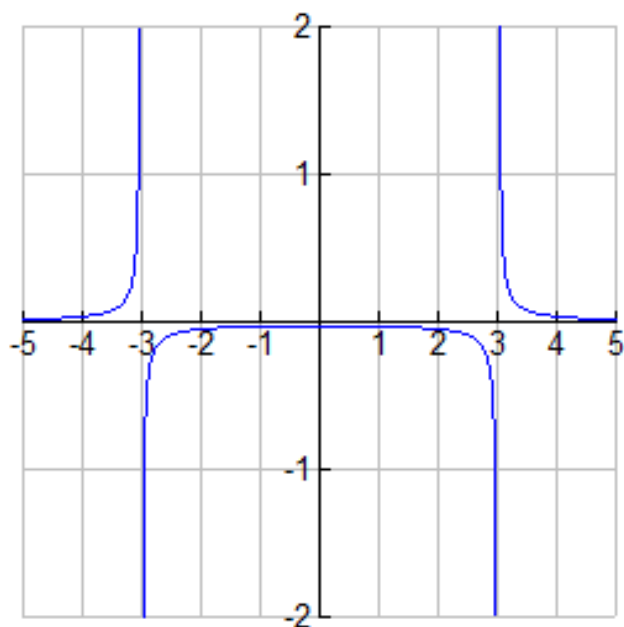
$y = \frac{1}{4}$

$$f(x) = \frac{(x^2 + 4x)}{(4x^2 - 16)}$$

Graph



$$3) f(x) = \frac{1}{4x^2 - 36}$$



Vertical Asym.: $x = 3, x = -3$

Holes: None

Horz. Asym.: $y = 0$

-2.

$$f(x) = \frac{1}{4x^2 - 36}$$

1. Simplify $\frac{1}{4(x^2 - 9)} = \frac{1}{4(x-3)(x+3)}$

2. Cancel out -- find the holes

Nothing cancels out

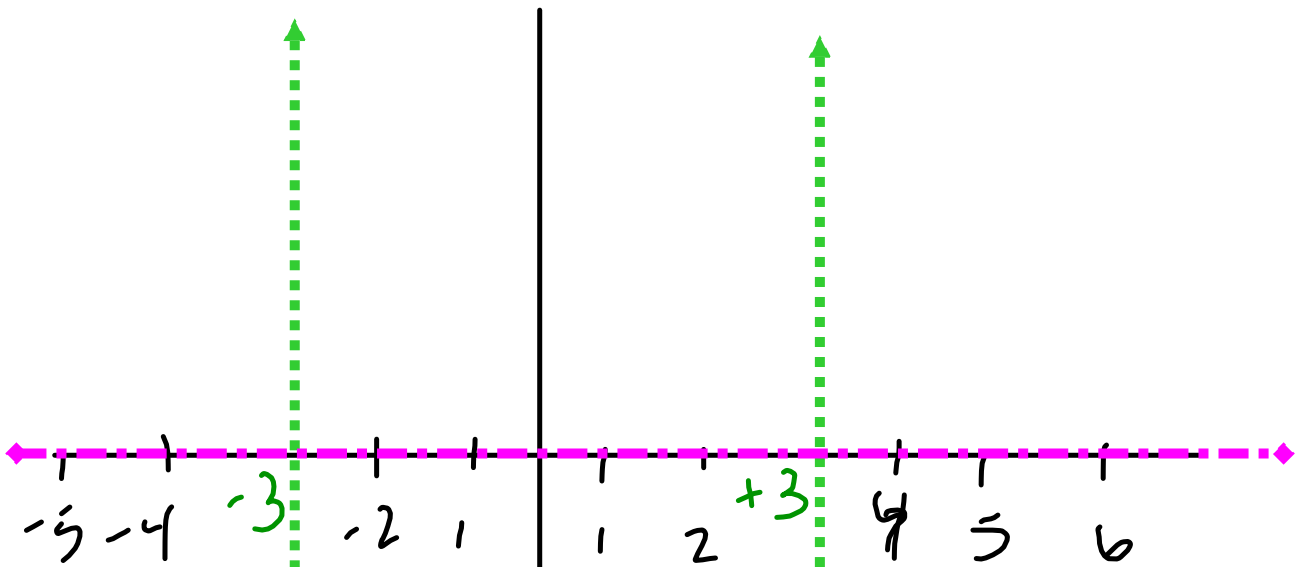
3. denominator = 0 -- vertical asymptotes

$$4(x-3)(x+3) = 0$$
$$x = 3 \quad x = -3$$

4. highest degree -- horizontal asymptote

$$\frac{1}{4x^2 - 36} \text{ bottom } y = 0$$

$$f(x) = \frac{1}{4x^2 - 36}$$



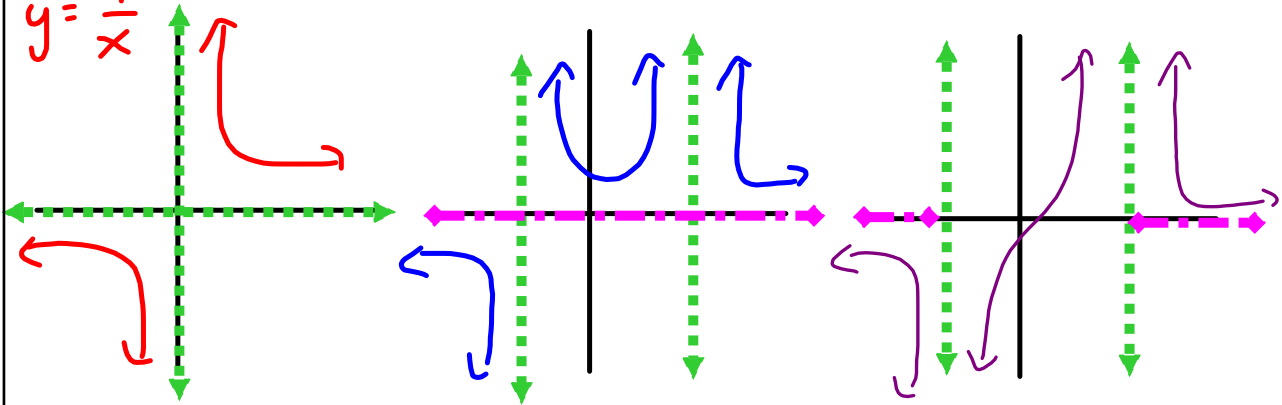
x	y
-4	
-5	
-6	

x	y
-1	
0	
1	

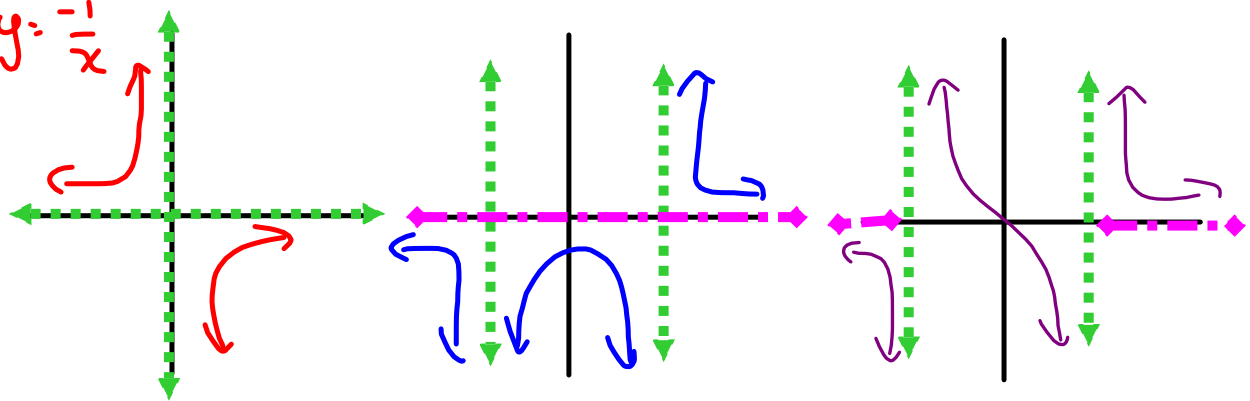
x	y
4	
5	
6	

What can the graphs look like?

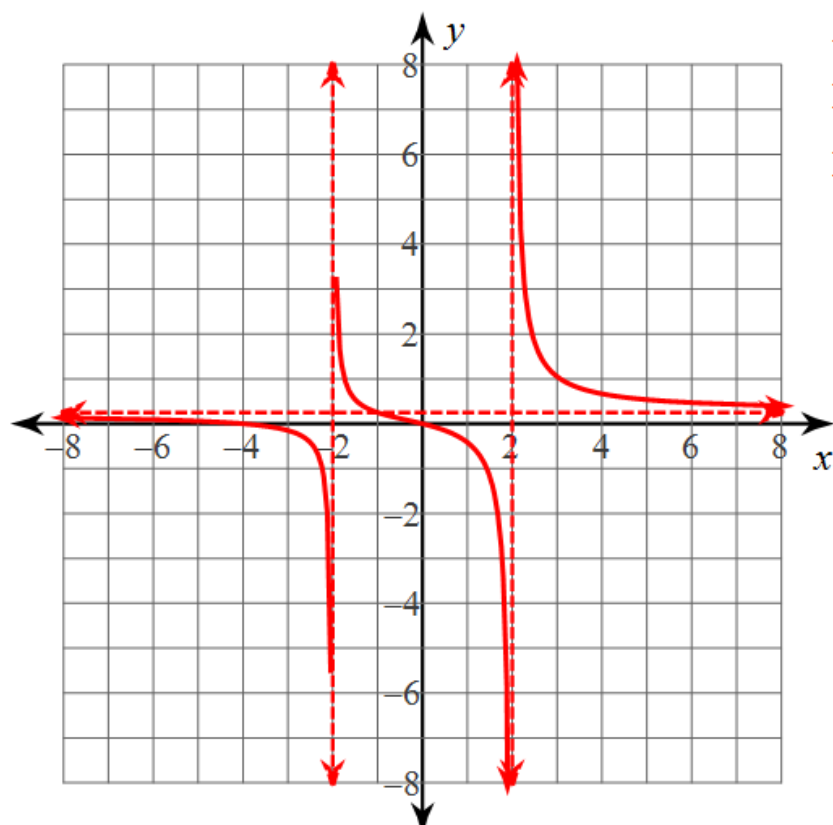
$$y = \frac{1}{x}$$



$$y = -\frac{1}{x}$$



$$2) f(x) = \frac{x^2 + 4x}{4x^2 - 16}$$



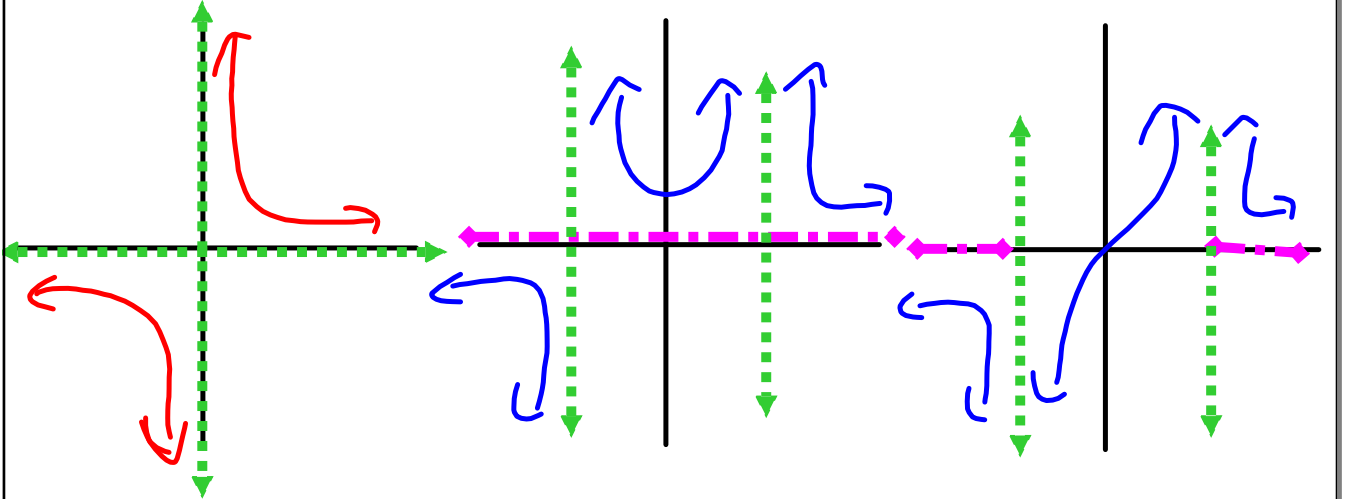
Vertical Asym.: $x = 2, x = -2$

Holes: None

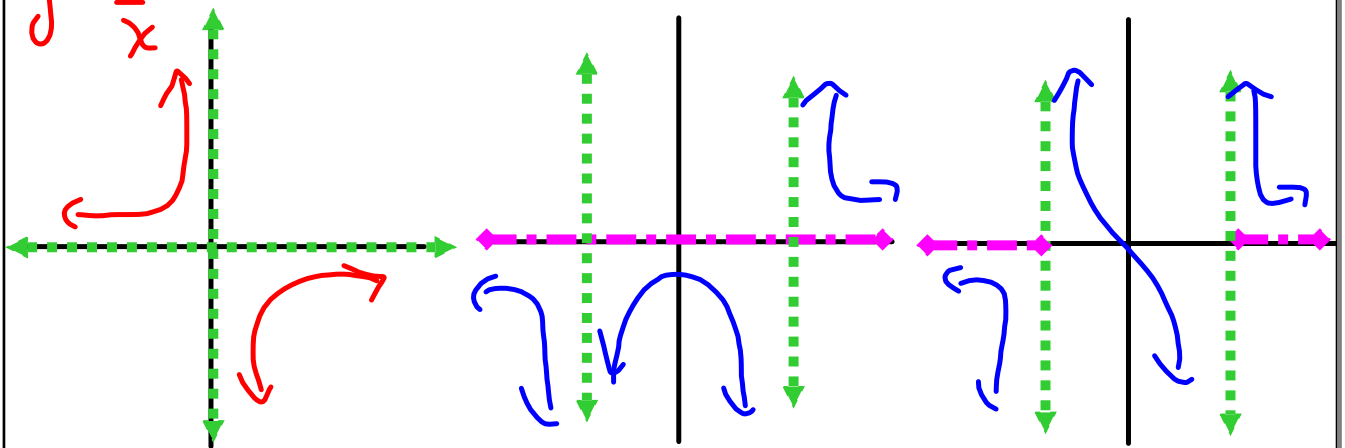
Horz. Asym.: $y = \frac{1}{4}$

What will the graphs look like?

$$y = \frac{1}{x}$$



$$y = -\frac{1}{x}$$



$$f(x) = \frac{x^2 + 4x}{4x^2 - 16}$$

1. Simplify
2. Cancel out -- find the holes
3. denominator = 0 -- vertical asymptotes
4. highest degree -- horizontal asymptote

$$f(x) = \frac{x^2 + 4x}{4x^2 - 16}$$

1. Simplify $\frac{x^2 + 4x}{4x^2 - 16} = \frac{x(x+4)}{4(x^2 - 4)} = \frac{x(x+4)}{4(x-2)(x+2)}$

2. Cancel out -- find the holes

$\frac{x(x+4)}{4(x-2)(x+2)}$ Nothing cancels out
 (No Holes)

3. denominator = 0 -- vertical asymptotes

$\frac{4(x-2)(x+2)}{4} = \frac{0}{4}$ $x = -2$ $x = 2$
 $(x-2)(x+2) = 0$
 $x-2=0$ $x+2=0$
 $x=2$ $x=-2$

4. highest degree -- horizontal asymptote

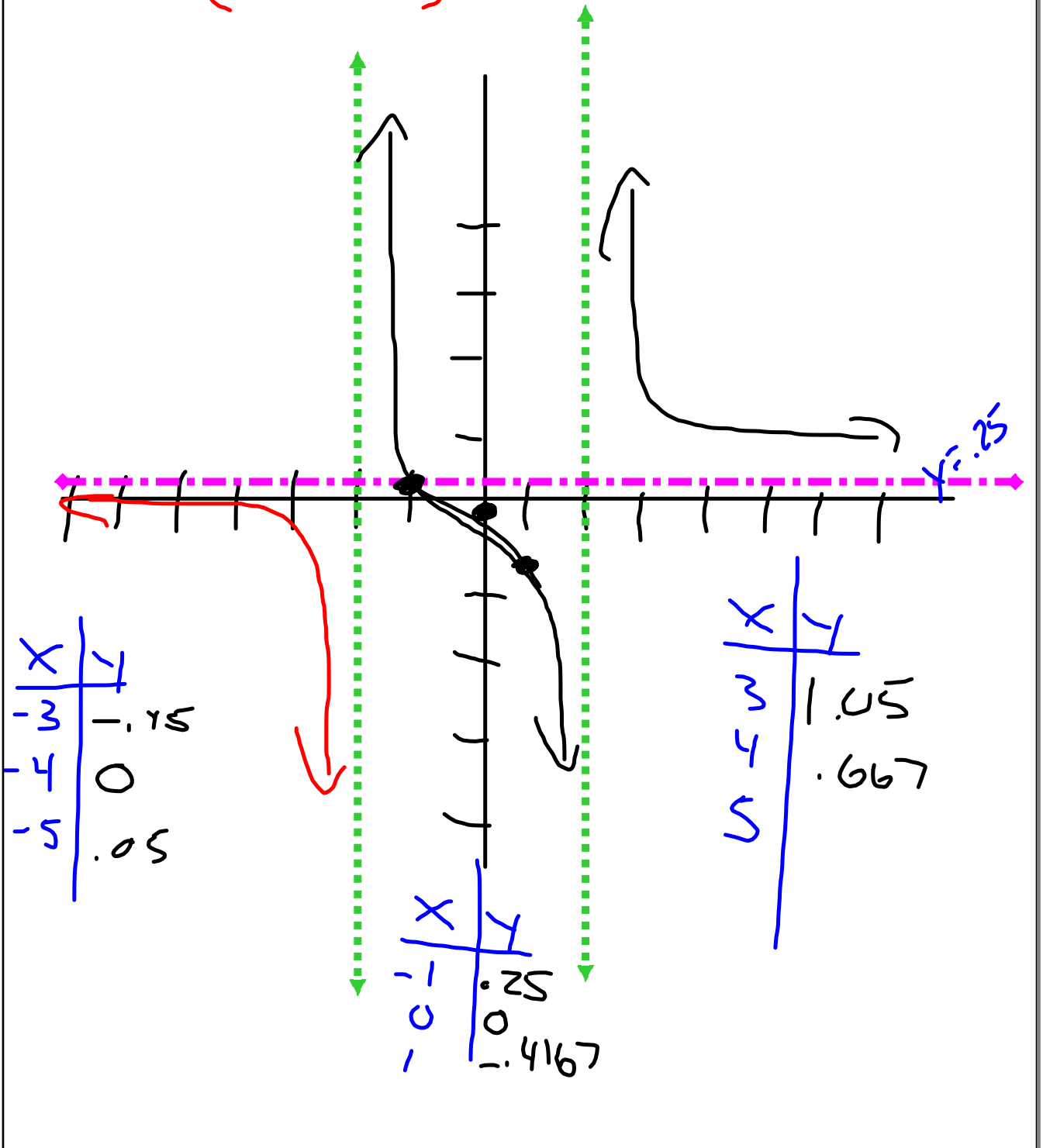
$f(x) = \frac{1x^2 + 4x}{4x^2 - 16}$
SAME
 coefficients of the highest degrees.

$y = \frac{1}{4}$

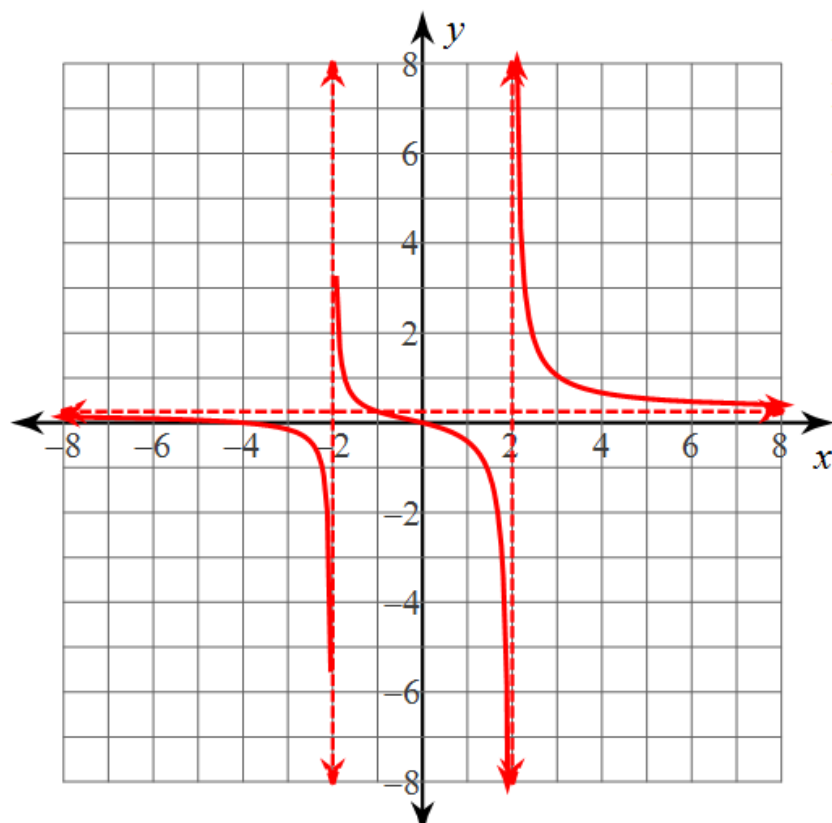
highest degree
 top slant
 bottom $y=0$

same $y = \frac{1}{4}$
 coefficients of highest degree

$$f(x) = \frac{(x^2 + 4x)}{(4x^2 - 16)}$$



$$2) f(x) = \frac{x^2 + 4x}{4x^2 - 16}$$



Vertical Asym.: $x = 2, x = -2$

Holes: None

Horz. Asym.: $y = \frac{1}{4}$

$$f(x) = \frac{1}{4x^2 - 36}$$

1. Simplify $\frac{1}{4(x^2 - 9)}$ = $\frac{1}{4(x-3)(x+3)}$

2. Cancel out -- find the holes

$\frac{1}{4(x-3)(x+3)}$ No holes

3. denominator = 0 -- vertical asymptotes

$\frac{1}{4(x-3)(x+3)}$ ($x-3$)($x+3$) = 0
x = 3 x = -3

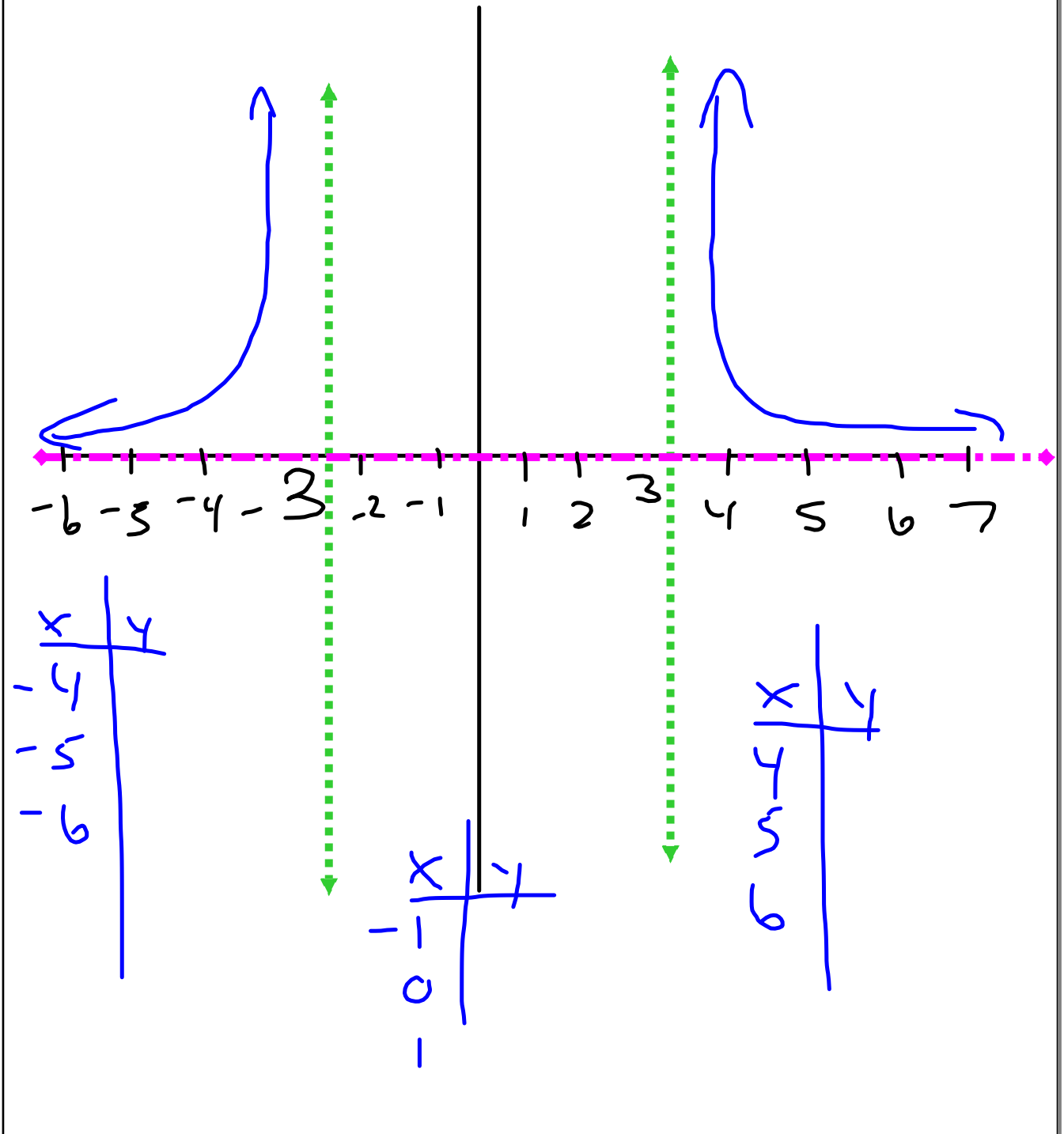
$$\frac{4(x-3)(x+3)}{4} = \frac{0}{4}$$

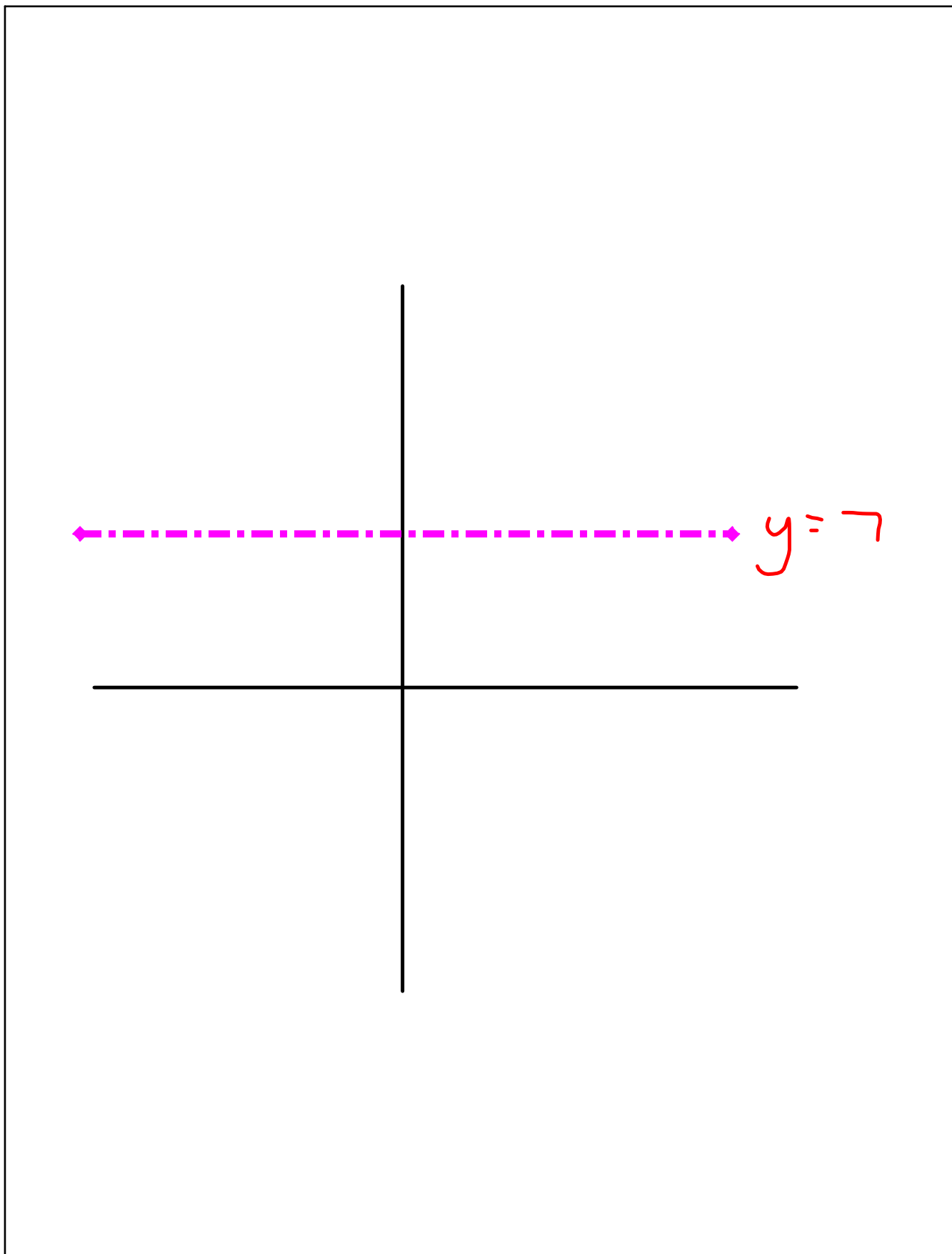
4. highest degree -- horizontal asymptote

$$f(x) = \frac{1}{4x^2 - 36}$$

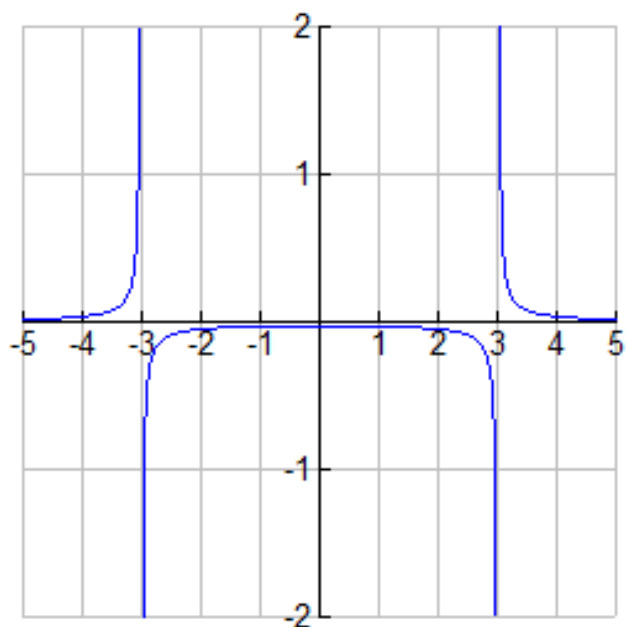
highest degree \uparrow bottom \Rightarrow Rules $y = 0$

$$f(x) = \frac{1}{4x^2 - 36}$$





$$3) f(x) = \frac{1}{4x^2 - 36}$$



Vertical Asym.: $x = 3, x = -3$

Holes: None

Horz. Asym.: $y = 0$

-2.