

Identify the points of discontinuity, holes, vertical asymptotes, x-intercepts, and horizontal asymptote of each.

1)  $f(x) = \frac{1}{3x^2 + 3x - 18}$   
 $3(x^2 + x - 6)$

2)  $f(x) = \frac{x-2}{x-4}$  *No holes*  
 VA:  $x = 4$   
 HA:  $y = 1$

3)  $f(x) = \frac{x^3 - x^2 - 6x}{-3x^2 - 3x + 18}$  HA: top slant

4)  $f(x) = \frac{x^2 + x - 6}{-4x^2 - 16x - 12}$

Holes: None  
 Restrictions

VA:

$-3(x+3)(x-2) = 0$

$\frac{-3(x+3)(x-2)}{-3} = \frac{0}{-3}$

$(x+3)(x-2) = 0$

$x+3=0 \quad x-2=0$

$x = -3 \quad x = 2$

$\frac{x(x^2 - x - 6)}{-3(x^2 + x - 6)} \quad \frac{x(x-3)(x+2)}{-3(x+3)(x-2)}$

4)  $f(x) = \frac{x^2 + x - 6}{-4x^2 - 16x - 12}$

HA: look at degree  
 SAME  $\frac{1x^2 \dots}{-4x^2 \dots} \Rightarrow y = -\frac{1}{4}$

$-1 \cdot 6$   $1 \cdot -6$   
 $-2 \cdot 3$   $2 \cdot -3$

Restrictions

Holes: ~~cancel~~  
 $\frac{(x-2)}{-4(x+1)} = \frac{-5}{8}$   $x+3=0$   
 $x = -3$   
 $(-3, -\frac{5}{8})$

$\frac{(x+3)(x-2)}{-4(x^2+4x+3)} = \frac{\cancel{(x+3)}(x-2)}{-4\cancel{(x+3)}(x+1)}$

↑  
 $1 \cdot 3$   
 $-1 \cdot -3$

VA:  $\frac{-4(x+1)}{-4} = 0$   
 $x+1=0$   
 $x=-1$

1)  $f(x) = \frac{1}{3x^2 + 3x - 18}$  ← HA: bottom  $y=0$

$3(x^2 + x - 6)$

$3(x+3)(x-2)$

FACTORS

-1 · 6	1 · -6
-2 · 3	2 · -3

$$f(x) = \frac{1}{3(x+3)(x-2)}$$

No holes → cancel out

$$\frac{\cancel{3}(x+3)(x-2)}{\cancel{3}} = \frac{0}{3}$$

VA:  $3(x+3)(x-2) = 0$

$x+3=0$     $x-2=0$

$x=-3$     $x=2$

Restrictions

5)  $f(x) = -\frac{4}{x^2 - 3x}$

HA: bottom  $y=0$

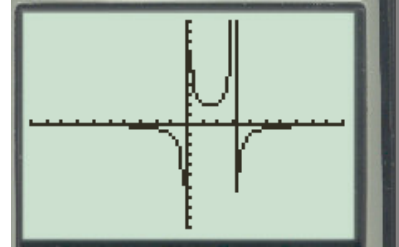
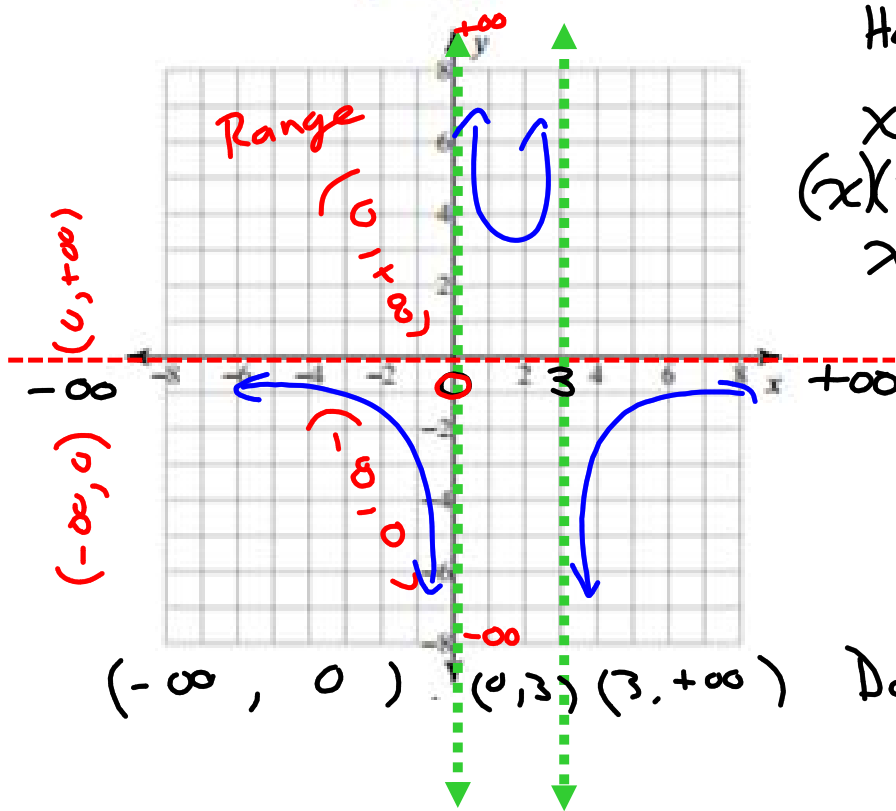
Holes & VA:

$$x^2 - 3x = 0$$

$$(x)(x - 3) = 0$$

$$x = 0 \quad x - 3 = 0$$

$$x = 3$$

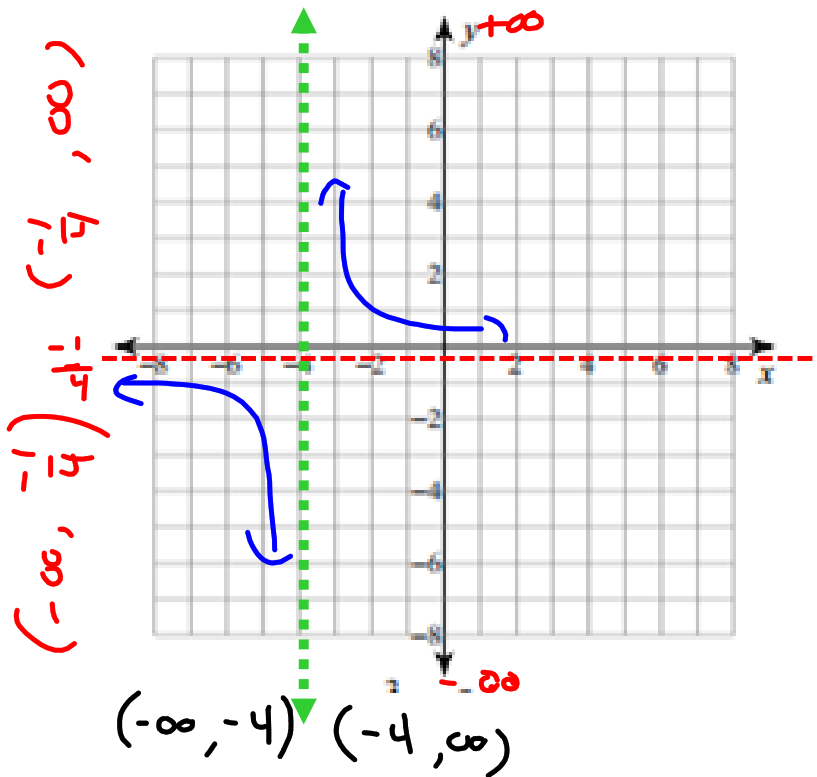


6)  $f(x) = \frac{x-4}{-4x-16}$

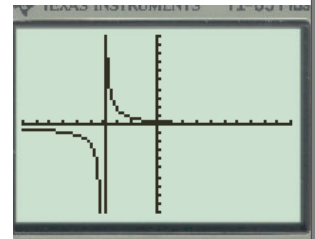
HA:  $y = -\frac{1}{4}$

Holes ~~None~~  
 $\frac{(x-4)}{-4(x+4)}$

VA:  
 $-4(x+4) = 0$   
 $(x+4) = 0$   
 $x = -4$



Range:  $(-\infty, -\frac{1}{4}) \cup (-\frac{1}{4}, \infty)$

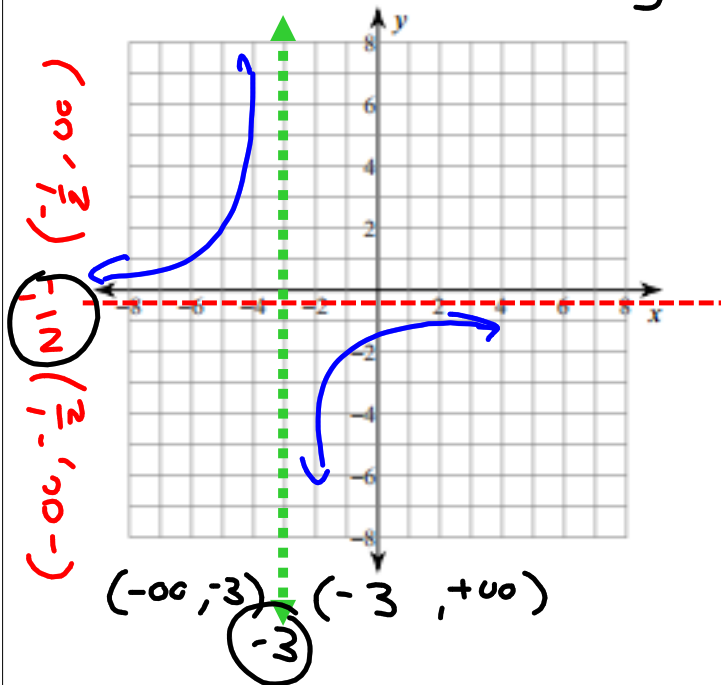


$$7) f(x) = \frac{x+4}{-2x-6}$$

HA: degrees are the same  
 $y = -\frac{1}{2}$

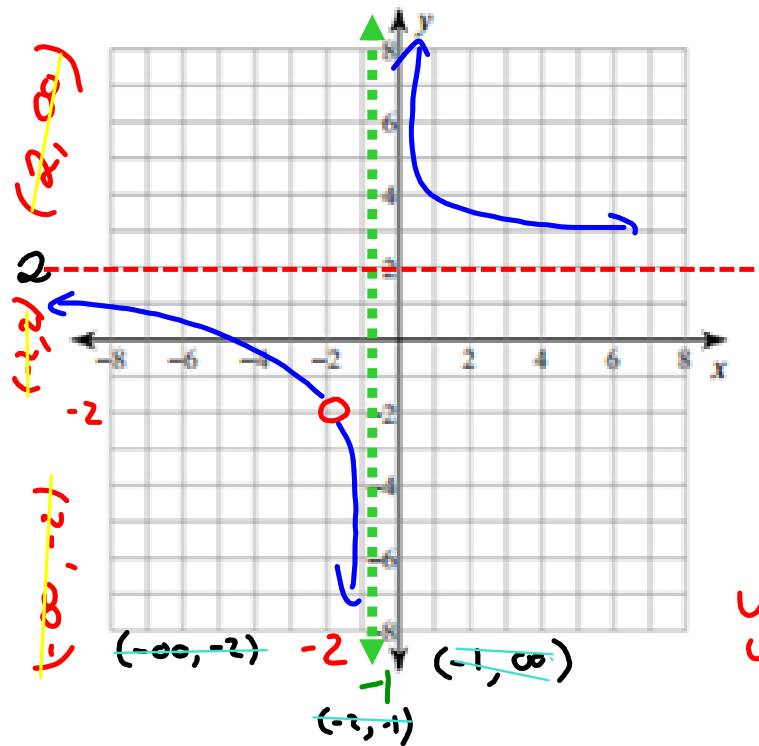
Holes: None  
 $\frac{x+4}{-2(x+3)}$

VA:  $-2(x+3) = 0$   
 $x = -3$



8)  $f(x) = \frac{2x^2 + 10x + 12}{x^2 + 3x + 2}$

HA:  $y = 2$



Holes:  $\frac{2(x^2 + 5x + 6)}{(x^2 + 3x + 2)}$

$\frac{2(x+3)(x+2)}{(x+1)(x+2)}$

Hole @  $x+2=0$   
 $x = -2$

$y = \frac{2(-2+3)}{(-2+1)} = \frac{2(1)}{-1} = -2$

$(-2, -2)$

VA:  $(x+1) = 0$   
 $x = -1$