A *rational expression* is the quotient of two polynomials.

Ex. 1 Find the domain of $g(x)=\frac{x^2-4x-21}{x^2-9x-20}$

Vertical Asymptote:

A vertical line that a graph approaches but never touches.

How to find:
Factor the denominator. Set all factors of the denominator (that are not factors of the numerator) equal to zero and solve.

Ex. 2 Identify all vertical asymptotes of the graph of $r(x)=\frac{3x}{x^2-3x+2}$

Horizontal Asymptote:

A horizontal line that a graph approaches but never touches.

How to find:
* If the degree of the numerator is less than the degree of the denominator, then y = 0 is the horizontal asymptote.

* If the degrees are equal, then the asymptote is y = a/b where a and b are leading coefficients.

* If the degree of numerator is greater than the degree of the denominator, then there is no horizontal asymptote.

Ex. 3 Identify all vertical and horizontal asymptotes of each of the following:

a. $R(x)=\frac{x^3}{x^2+x-20}$

b. $H(x)=\frac{2x^2-2x+1}{x^2-x-12}$

Holes in Graphs

After factoring the numerator and denominator of the function, if something is a factor of both the numerator and denominator, then there is a hole in the graph.

Ex 4 : Let $y=\frac{3x^2+x^3}{x^2+2x-3}$. Identify all asymptotes and holes in the graph.

Ex 5: Let $y=\frac{3x-28+x^2}{x^2+12x+35}$. Identify all asymptotes and holes in the graph.